Special Relativity

1) The average lifetime of a muon is $\tau_\mu = 2.2\times10^{-6}s$, and its mass is $m_\mu = 0.105$ GeV$/c^2$. What is the average distance travelled by the particle in a frame where its energy is 1 GeV?

2) Show that if a final state of a set of N particles is split into 2 subsets C_1 and C_2, with N_1 and N_2 particles ($N = N_1 + N_2$), the energy of the subset C_1 in the proper frame of the initial state is:

$$E_{C_1} = \frac{M^2 + M_1^2 - M_2^2}{2 \cdot M} \tag{1}$$

where M_i is the invariant mass of the subset C_i, and M is the total invariant mass of the final state.

Show that the momentum of the subset C_1 in the same frame is:

$$P_{C_1} = \frac{\sqrt{(M^2 - (M_1 + M_2)^2) \cdot (M^2 - (M_1 - M_2)^2)}}{2 \cdot M} \tag{2}$$

N.B.: These formulas may be applied in the specific case of a two body decay.

3) Consider a π^- beam of energy 100 GeV in the laboratory frame. We look at the decay $\pi^- \to \mu^- \bar{\nu}_\mu$. Determine the minimum and maximum energies E_{min} and E_{max} of the anti-neutrinos in the laboratory frame. Justify that the decay is isotropic and determine the energy spectrum of the neutrinos.

4) We consider a system of invariant mass M_0 to which is added an extra particle of mass m. Show that the invariant mass of the resulting system M' is equal or larger than the sum $M_0 + m$.

Deduce the maximum energy of a particle produced in a $N \geq 3$ body decay. Then, what is the configuration of the $N - 1$ other particles ?

Inversely, what is the minimum energy of a particle in a $N \geq 3$ body decay ?

5) Let's consider a reaction $1 + 2 \to X$ with a given invariant mass M_{12} and the two following frames :
R^*: frame of the centre of mass (proper frame of the system $1 + 2$)
R^{lab}: laboratory frame, where the particle 2 is at rest ("target"). Express the energy E_1^{lab} with respect to E_1^* in the ultra-relativistic limit.

The HERA collider at Hamburg produced collisions between electrons of 30 GeV and protons of 820 GeV. What energy should have an electron beam colliding a fixed target to explore the same invariant mass domain M_{ep} ?