

2. Linearized GL eq^s : nucleation of superconductivity.

• Linearized GL eq^s (Gross-Pitaevski):

$$\left\{ \frac{1}{2m} \left(\frac{\hbar}{i} \vec{\nabla} - e \vec{A} \right)^2 + \alpha \right\} \psi = 0$$

(dropping $|\psi|^4$ term, i.e. d.o.p. down to crit. pt where o.p. v.o.).

$$\left(\frac{\vec{\nabla}}{i} - \frac{e \vec{A}}{\hbar} \right)^2 \psi = - \frac{2m\alpha}{\hbar^2} \psi = \frac{\psi}{\xi^2(\tau)} \quad (*)$$

→ analogous to Schrödinger eq w. eigenvalue $\frac{1}{\xi^2}$.

$$\odot \vec{A} = \vec{A}_{\text{ext}} : \vec{J}_s = \frac{ke}{2m} \left(\psi^* \vec{\nabla} \psi - \psi \vec{\nabla}^* \psi^* \right) \propto |\psi|^2$$

⇒ dropped in linearizatⁿ.

≡ forget all screening current effects.

$$1) \quad \frac{\beta}{2} |\psi|^4 \ll |\alpha| |\psi|^2 = \beta |\psi_0|^2 |\psi|^2 \quad |\psi_0|^2 = \frac{-\alpha}{\beta}$$

$$|\psi|^2 \ll 2 |\psi_0|^2.$$

$$2) \quad \text{Landau gauge: } \vec{H} \parallel \vec{u}_z \quad \vec{A} = Bx \vec{u}_y.$$

$$\psi = e^{ik_y y} e^{ik_z z} f(x).$$

$$\begin{aligned} (*) \rightarrow \frac{1}{\xi^2} \psi &= \left((-i\vec{\nabla})^2 + \frac{e^2 A^2}{\hbar^2} + 2i\vec{\nabla} \cdot \frac{e \vec{A}}{\hbar} \right) \psi \\ &= -\Delta \psi + \frac{e^2 A^2}{\hbar^2} \psi + 2i \frac{e \vec{A}}{\hbar} \cdot \vec{\nabla} \psi \quad \leftarrow \parallel \vec{u}_y \\ &= (k_y^2 + k_z^2) \psi + e^{ik_y y} e^{ik_z z} f''(x) + \frac{e^2 A^2}{\hbar^2} \psi + \frac{2e \vec{A}}{\hbar} k_y \psi \\ &= f''(x) + \frac{e^2 B^2 x^2}{\hbar^2} f - \frac{2eBx}{\hbar} k_y f + k_y^2 f = \left(\frac{1}{\xi^2} - k_z^2 \right) f(x) \end{aligned}$$

$$\left| -\frac{1}{2} \left(\frac{eB}{\hbar} \right)^2 (x - x_0)^2 \right| = \left(\frac{1}{\xi^2} - k_z^2 \right) \frac{\hbar^2}{2m} \quad (**)$$

$x_0 = \frac{\hbar k_y}{eB}$

$$2) \quad (**) \times \frac{\hbar^2}{2m} \rightarrow -\frac{\hbar^2}{2m} \frac{1}{2} \left(\frac{eB}{m} \right)^2 (x - x_0)^2 = \frac{\hbar^2}{2m} \left(\frac{1}{\xi^2} - k_z^2 \right) \frac{\hbar^2}{2m}$$

$\omega_c^2 = \text{cyclotron freq.}$

3) $\Rightarrow$ eq. of a 1D harmonic oscillator

part. w. const momentum $\hbar k_z \parallel \vec{B}$ that moves in a circle in (x, y) .

spectrum: $\hbar \omega_c (n + 1/2) \equiv$ Landau levels. formally equiv. to that of normal charged part in a mag. field

$$4) \quad \text{Energies: } E(k_z, n) = \hbar \omega_c (n + 1/2) - \frac{\hbar^2}{2m} \left(\frac{1}{\xi^2} - k_z^2 \right)$$

$$\text{And: } \hbar \omega_c (n + 1/2) = \left(\frac{1}{\xi^2} - k_z^2 \right) \frac{\hbar^2}{2m}$$

$$\frac{\hbar e B}{m} (n + 1/2) = \frac{\hbar^2}{2m} \left(\frac{1}{\xi^2} - k_z^2 \right)$$

$$\phi_0 = \frac{h}{2e} = \text{quantum of flux.}$$

$$= \frac{h}{e}$$

$$\boxed{H = \frac{B}{\mu_0} = \frac{\phi_0}{2\mu_0 2\pi (n + 1/2)} \left(\frac{1}{\xi^2} - k_z^2 \right)}$$

max for $n=0, k_z=0$.

$$\boxed{H_{c2} = \frac{\phi_0}{\mu_0 2\pi \xi^2}}$$

highest field at which SC nucleates in the bulk

See also Aladyshkin's lecture.

5) If $\psi(x) = \exp \left[-\frac{(x-x_0)^2}{2\xi^2} \right]$ Δ here $k_z, \mu = 0$.

$$\psi'(x) = -\frac{(x-x_0)}{\xi^2} \exp \left[\right]$$

$$\psi''(x) = \left\{ \frac{1}{\xi^2} + \frac{(x-x_0)^2}{\xi^4} \right\} \exp \left[\right]$$

$$(*) \rightarrow \frac{1}{\xi^2} = \frac{(x-x_0)^2}{\xi^4} + \left(\frac{eB}{\hbar} \right)^2 (x-x_0)^2 = \frac{1}{\xi^2} - k_z^2$$

$$\frac{\hbar \omega_c}{\hbar^2} \frac{1}{2} = \frac{\omega_c m}{\hbar}$$

we have $\frac{1}{2} \hbar \omega_c = \frac{\hbar^2}{2m} \left(\frac{1}{\xi^2} \right)$

$$\frac{1}{\xi^2} = \frac{m \omega_c}{\hbar}$$

and $\frac{eB}{\hbar} = \frac{m \omega_c}{\hbar} \leftarrow \frac{eB}{m}$

so that: $\frac{m \omega_c}{\hbar} - \left(\frac{m \omega_c}{\hbar} \right)^2 (x-x_0)^2 + \left(\frac{m \omega_c}{\hbar} \right)^2 (x-x_0)^2 = \frac{m \omega_c}{\hbar}$

ok $\Rightarrow \psi(x)$ has this

$$\Rightarrow \psi = e^{iky} e^{ik_z z} e^{-\frac{(x-x_0)^2}{2\xi^2}}$$

exp.
usual wavefunction
for 1D H.O.

for $k_z, \mu = 0$.

6) $H_{c2} = \frac{\hbar}{e_x \mu_0 \xi^2} = \frac{-2m\alpha}{e_x \mu_0 \hbar}$

$$\frac{1}{2} \mu_0 H_c^2 = \frac{1}{2} \mu_0 - \frac{1}{2} \mu_0 = - \left(\alpha |4\pi|^2 + \frac{\beta}{2} |4\pi|^4 \right) = - \left(-\frac{\alpha^2}{\beta} + \frac{\beta}{2} \frac{\alpha^2}{\beta^2} \right)$$

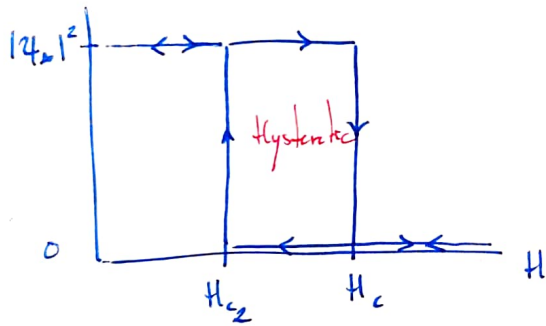
$$= \frac{\alpha^2}{2\beta}$$

$$\lambda_L^2 = \frac{\mu_0}{\mu_0 \mu_s e_s^2} = \frac{\mu_s}{\mu_0 |4\pi|^2 e_s^2} = \frac{-\mu_s \beta}{\mu_0 \alpha e_s^2}$$

$$\begin{aligned}
 k^2 &= \frac{j^2}{\xi^2} = \frac{-m\beta}{\mu_0 d e^2} \frac{H_{c2} e \mu_0}{h} = \frac{-m\beta}{h d e} H_{c2} \\
 &= \frac{-m}{h e} \frac{d}{2} \frac{2}{\mu_0 H_{c2}} H_{c2} \quad \alpha = -\frac{e \mu_0 h}{2 dm} H_{c2} \\
 &= \frac{-m}{h e} \frac{e \mu_0 h}{2 dm} H_{c2} \frac{1}{\mu_0 H_{c2}} H_{c2} \\
 k^2 &= \frac{H_{c2}^2}{2 H_c^2}
 \end{aligned}$$

$$\sqrt{2} k H_c = H_{c2}$$

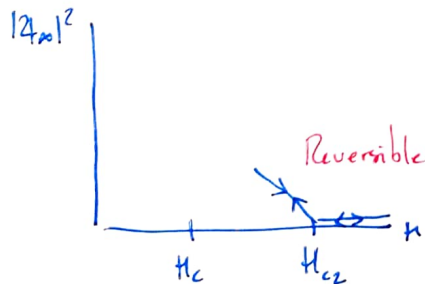
- Type I SC: $k < 1/\sqrt{2}$ $\rightarrow H_{c2} < H_c$.



- H_{c2} : nucleat: only possible @ H_{c2} where $|4\pi M|^2$ jumps (1st order transit: to $|4\pi M|^2$.

- Type I SC "supercool" and remain normal even below H_c .

- Type II SC: $k > 1/\sqrt{2}$ $\rightarrow H_{c2} > H_c$.



- 2nd order transit: @ $H_{c2} > H_c$