

Vortex in superconductors

The diagram illustrates the vortex state in superconductors across different magnetic field ranges:

- Normal state:** Represented by a blue circle labeled "Normal state" with vertical arrows passing through it, corresponding to the magnetic field range $B > B_{C2}$.
- Superc. state:** Represented by a blue circle labeled "Superc." with red lines indicating superconducting regions, corresponding to the magnetic field range $B < B_{C1}$.
- Vortex lattice:** A detailed view of the vortex lattice in the intermediate field range $B_{C1} < B < B_{C2}$. It shows a grid of vortices (black dots) with red arrows indicating the direction of the magnetic field lines. A red arrow points to the "Vortex lattice" label.

Portraits of three scientists are shown in the top right, with the middle one circled in red and labeled **Alexei Abrikosov**.

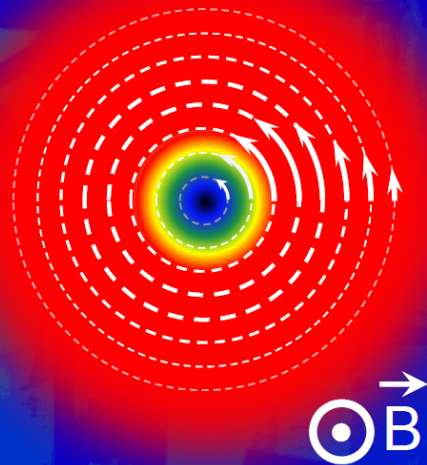
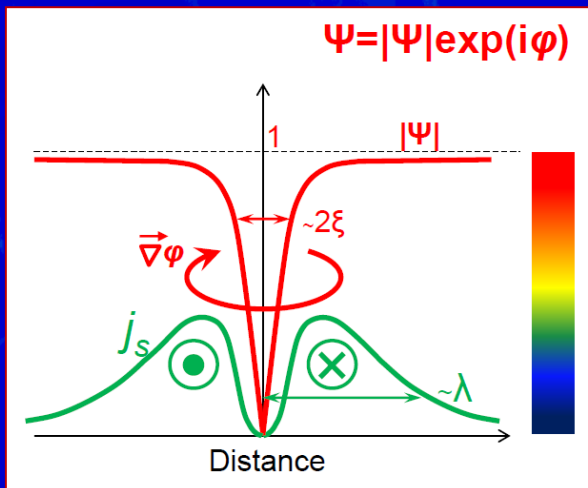
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Rotating Quantum Condensates: Vortex Matter

$$\frac{1}{2m^*}(-i\hbar\vec{\nabla} - 2e\vec{A})^2\Psi + \beta|\Psi|^2\Psi = -\alpha\Psi$$

$$\vec{j} = \frac{2e}{m^*}|\Psi|^2(\hbar\vec{\nabla}\varphi - 2e\vec{A}) = 2e|\Psi|^2\vec{v}_s$$

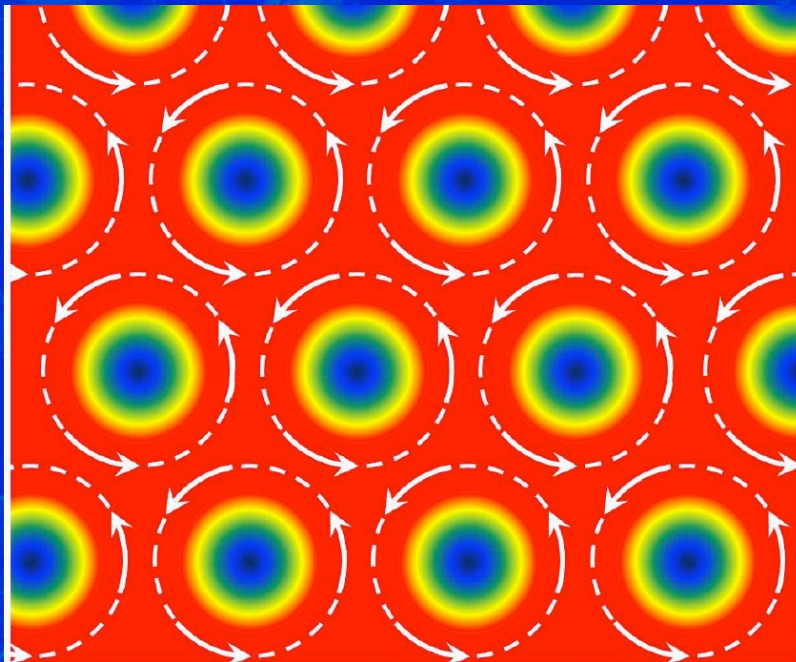
GL theory
+
Abrikosov



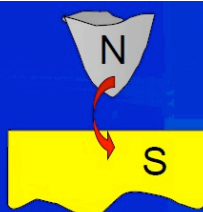
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Rotating Quantum Condensates: Vortex Matter

Drawing of the vortex lattice (Abrikosov, Nobel 2003)



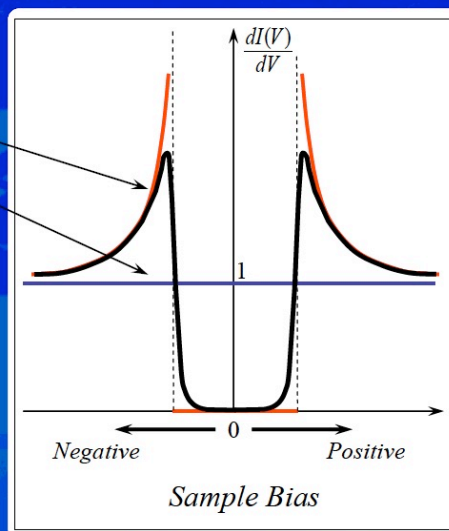
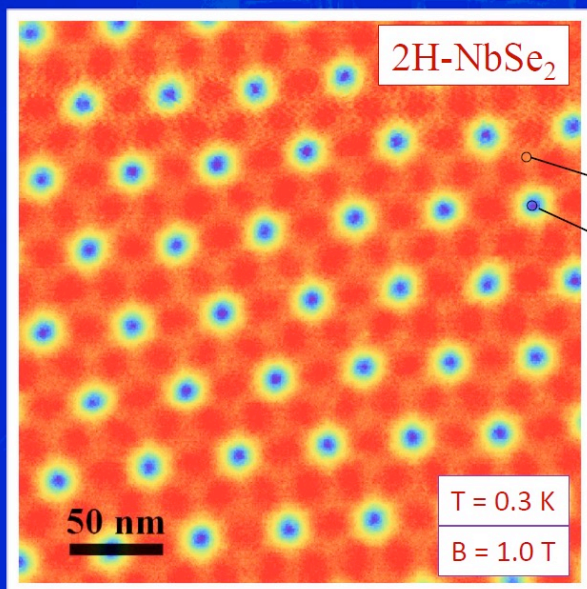
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Scanning Tunneling Spectroscopy of vortices

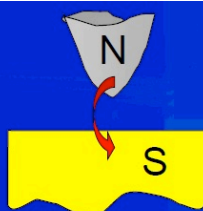
$$\frac{dI}{dV}(V) = -\frac{2\pi e^2 T}{\hbar} N_p \int_{-\infty}^{\infty} N_s(E+eV) \frac{\partial f}{\partial E}(E) dE$$

Experiment: Visualization of the vortex cores by STS



D. Roditchev et al, Paris, to be published

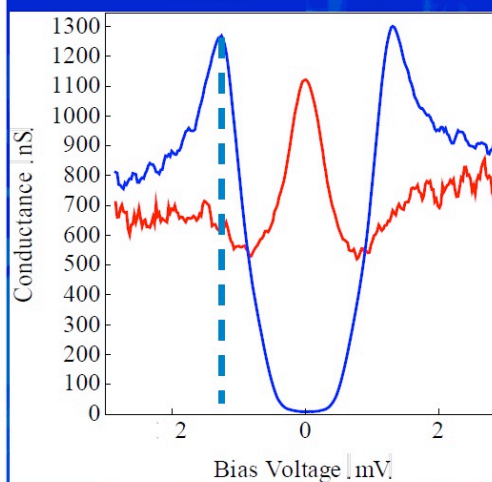
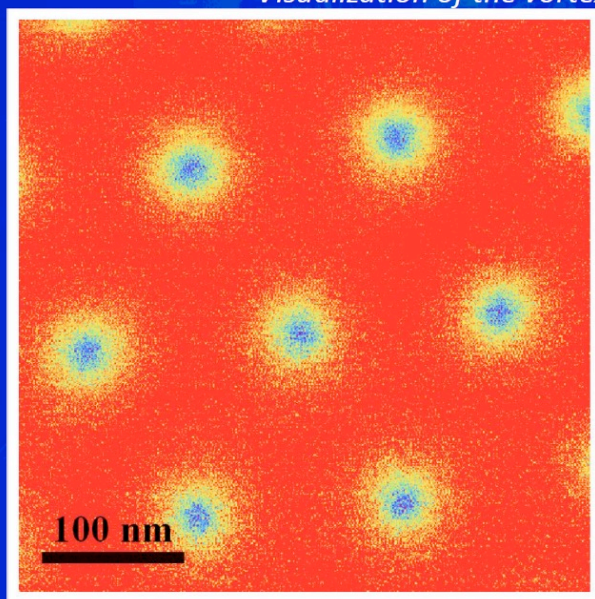
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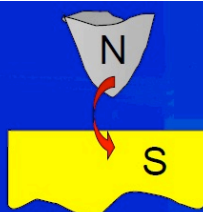
Scanning Tunneling Spectroscopy of vortices

$$\frac{dI}{dV}(V) = -\frac{2\pi e^2 T}{\hbar} N_p \int_{-\infty}^{\infty} N_s(E+eV) \frac{\partial f}{\partial E}(E) dE$$

Visualization of the vortex cores at Gap Energy



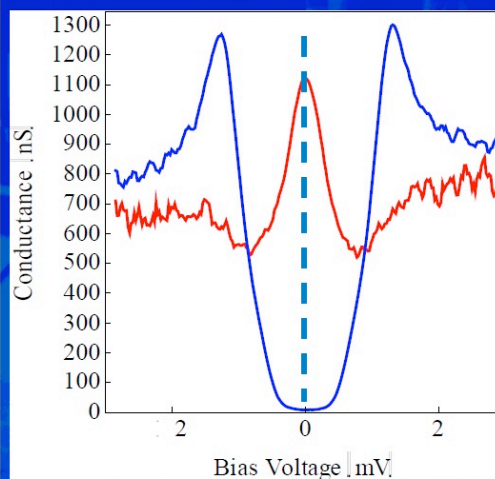
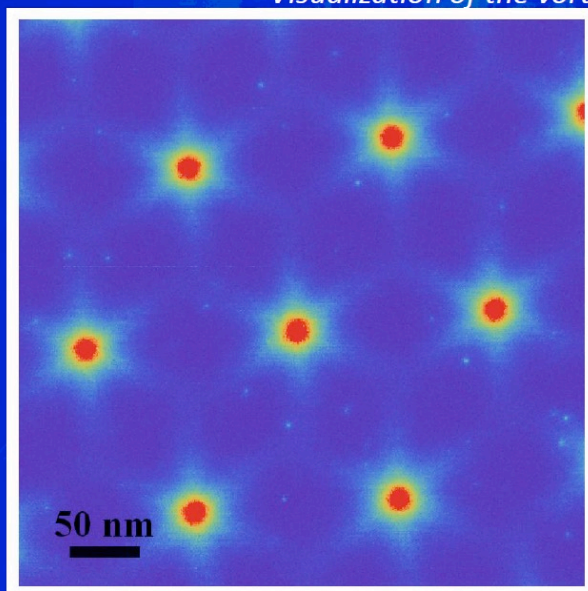
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Scanning Tunneling Spectroscopy of vortices

$$\frac{dI}{dV}(V) = -\frac{2\pi e^2 T}{\hbar} N_p \int_{-\infty}^{\infty} N_s(E+eV) \frac{\partial f}{\partial E}(E) dE$$

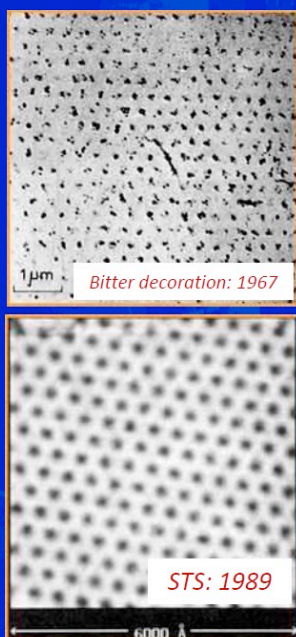
Visualization of the vortex cores at **Zero Bias**



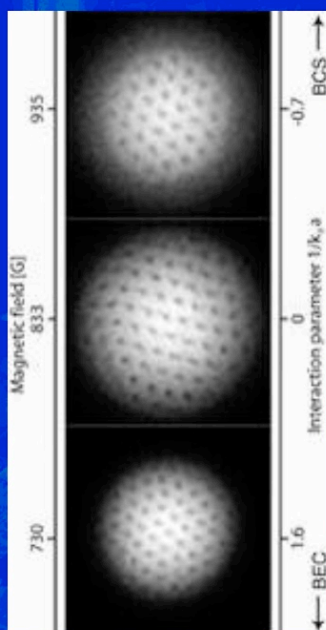
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Rotating Quantum Condensates: Vortex Matter

Superconductors (BCS)

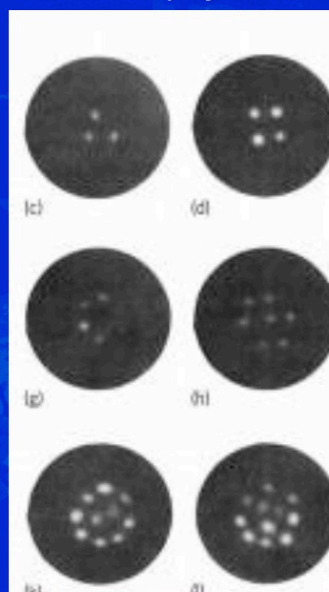


Ultracold atoms (BEC)



Quantum superfluids

Vortex in suprefluid He⁴



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Our motivation: Phase Diagram of Confined Superconductors

Type II superconductors ($K = \lambda/\xi > 1$)

$$D \sim \xi$$

Size of the vortex cores comparable to the sample size $\longrightarrow$ **Strong Confinement**

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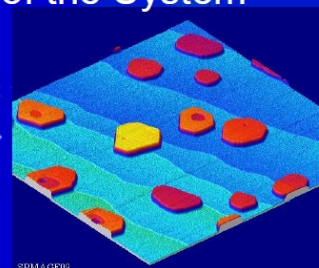
Strong Confinement Regime: Choice of the System

Bulk Pb ($\xi_0 = 80\text{nm}$, $\lambda_0 = 50\text{nm}$)

Our case: disordered Pb/Si interface $\longrightarrow$

Mean free path l :

$$l \approx 2\hbar = 2 \times 5.5\text{nm} = 11\text{nm} \ll \xi_0 \quad \text{Dirty limit SC}$$

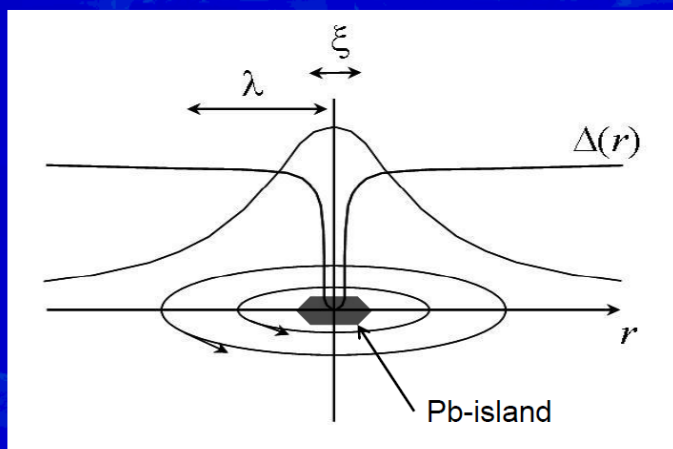


$$h \ll D = 110\text{nm}$$

$$\xi_{\text{EFF}} \approx 40\text{nm} \approx 0.3D$$

$$\lambda_{\text{EFF}} \approx 150\text{nm} \approx D$$

$$\Lambda \approx 4000\text{nm} \approx 40D$$



Result: our Pb-island is the type II dirty limit SC : Magnetic field fully penetrates ($\Lambda \gg D$), flux is not quantized.

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Our motivation: Phase Diagram of Confined Superconductors

Strong Type II ($K = \lambda/\xi \gg 1$):

$$D \sim \xi, \xi \ll \lambda$$

$$D \ll \lambda$$

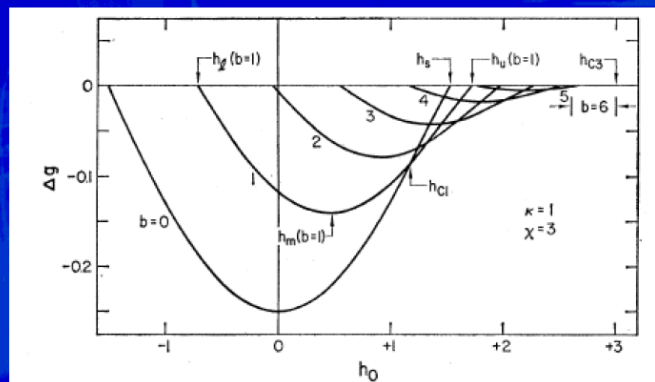
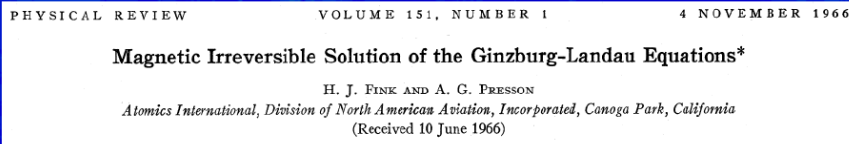


- tiny magnetic response,
- variations at nanometer scale

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Confined Vortex Configurations: Our Motivations

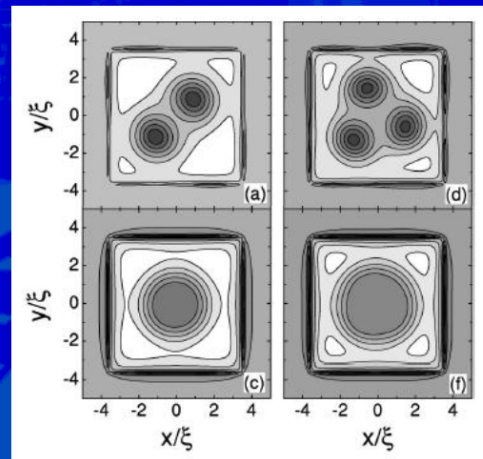
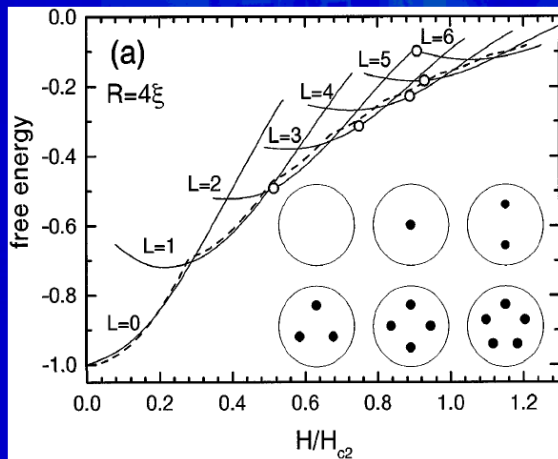
Phase Diagram of Confined Superconductors



$$\vec{j} = \frac{2e}{m^*} |\Psi|^2 (\hbar \vec{\nabla} \varphi - 2e \vec{A}) = 2e |\Psi|^2 \vec{v}_s$$

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Phase Diagram of Confined Superconductors Giant Vortex States



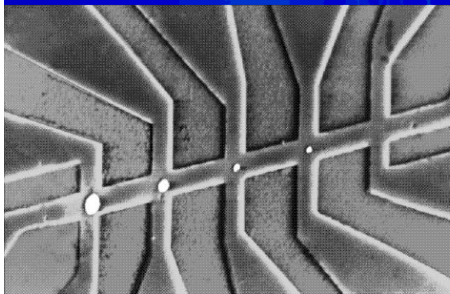
Superconducting nano-islands having a size of $\sim \xi$ should have peculiar properties due to the lateral confinement.

V. Schweigert et al., *Phys. Rev. Lett.* 81, 2783 (1998)

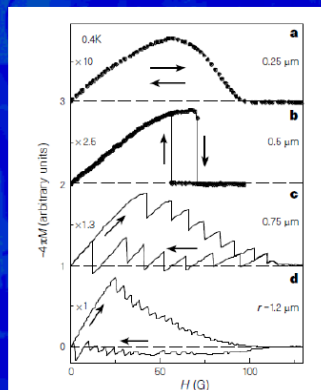
B. Baelus and F. Peeters, *Phys. Rev. B* 65, 104515 (2002)

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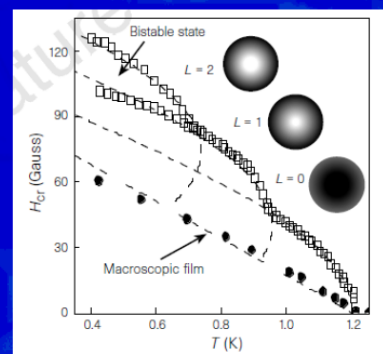
Detection of magnetic vortex states by Hamm magnetometry



Hall magnetometry of
mesoscopic Al
superconducting disks



Detection of individual
quantum flux penetration



Giant vortex State?

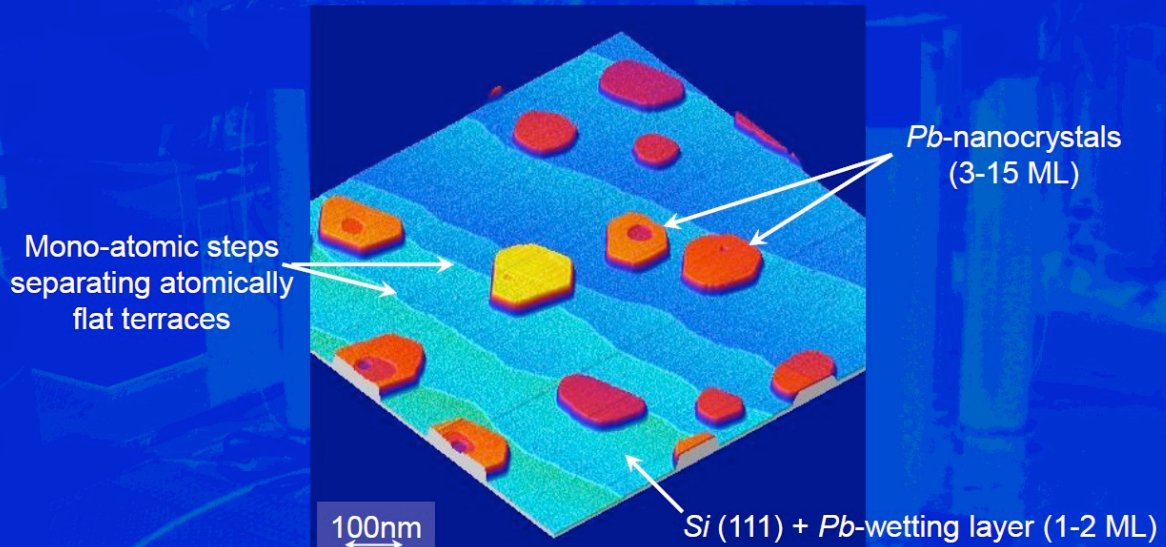
Phase transitions in individual sub-micrometre Superconductors,
A. Geim et al. *Nature*, 390, 259 (1997)

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Response of Confined Superconducting Condensate to an External Magnetic Field

Samples:

in-situ grown Pb-islands on 7x7 reconstructed Si(111)

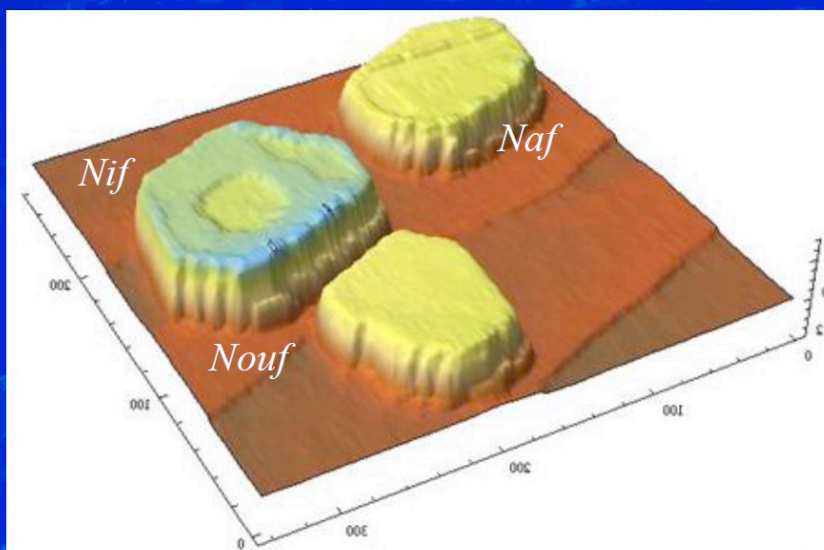


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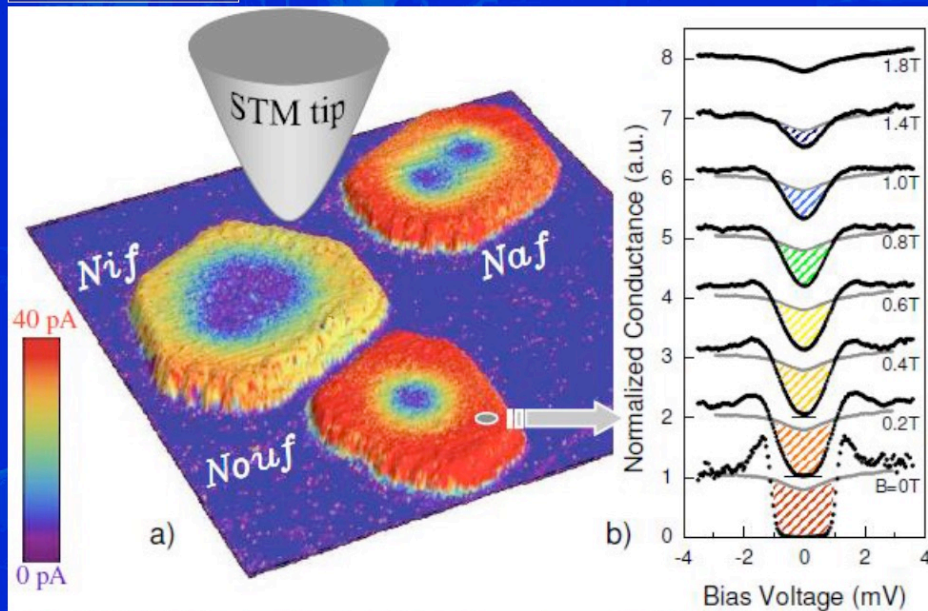
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Strongly Confined Superconducting Condensate

STS: G.A. maps

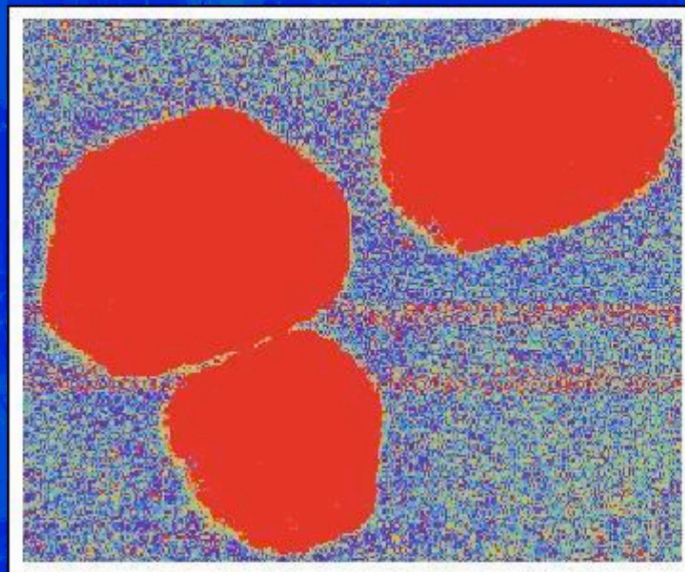
0.8T : 10 times H_c (bulk Pb)

0.3K ($T/T_c=1/20$)



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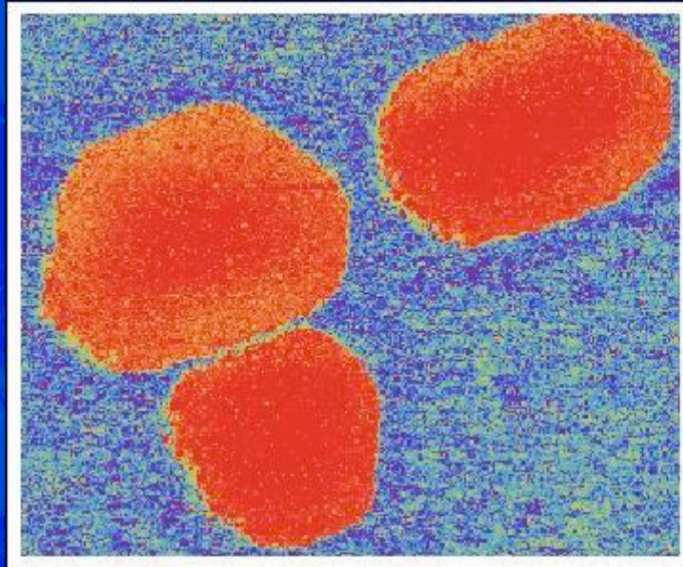
Strongly Confined Superconducting Condensate



Homogeneous superconductivity at
B=0T & T=280mK

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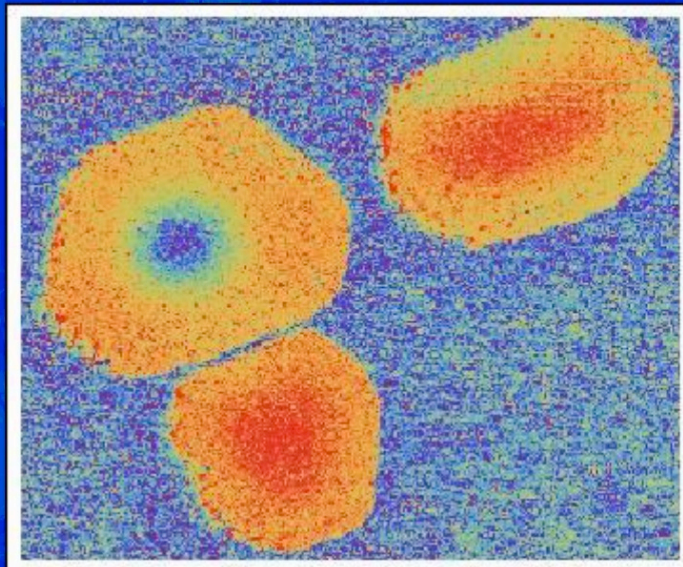
Strongly Confined Superconducting Condensate



Meissner state at $B=0.2\text{T}$ &
 $T=280\text{mK}$

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Strongly Confined Superconducting Condensate



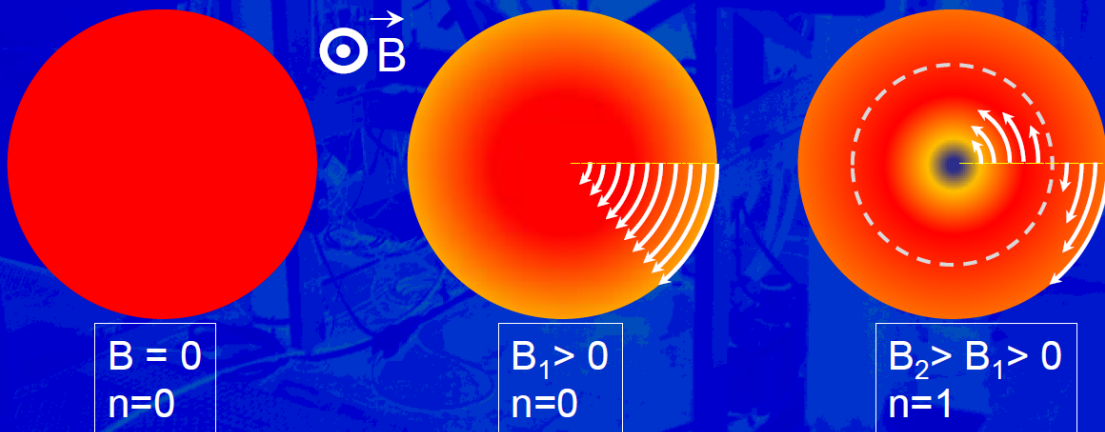
Meissner states and one vortex
state at $B=0.4\text{T}$

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How should we understand STS data?

$$\vec{A} = \frac{1}{2} \vec{B} \wedge \vec{r}$$

$$\vec{j} = \frac{2e}{m^*} |\Psi|^2 (\hbar \vec{\nabla} \varphi - 2e \vec{A})$$

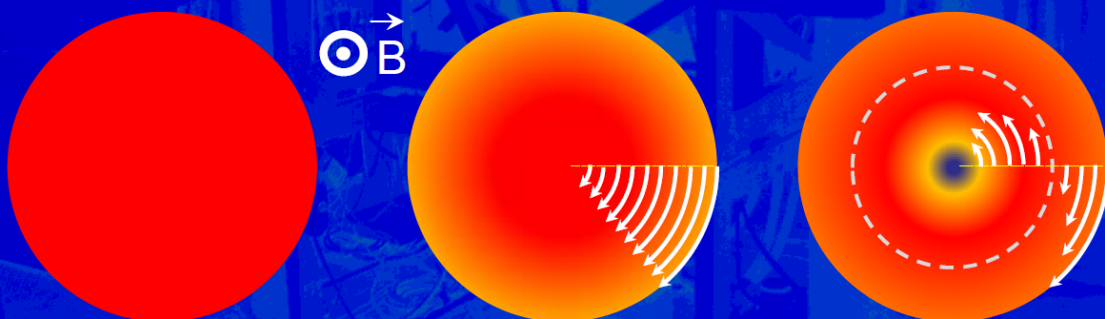


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$$\vec{A} = \frac{1}{2} \vec{B} \wedge \vec{r}$$

$$\vec{j} = \frac{2e}{m^*} |\Psi|^2 (\hbar \vec{\nabla} \varphi - 2e \vec{A})$$



Kinetic energy $E_{\text{currents}} \approx n_s m^* \langle v_s^2 \rangle / 2 \sim j^2 \sim B^2$

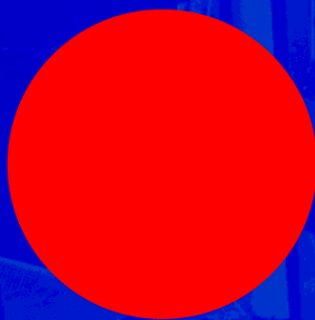
Radial density $\partial E_{\text{currents}} / \partial \rho \sim B^2 \rho^3$

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How should we understand STS data?

$$\vec{A} = \frac{1}{2} \vec{B} \wedge \vec{r}$$

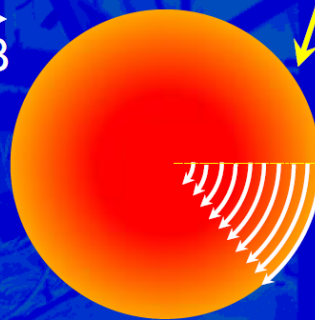
$$\Phi + \frac{1}{2e} \oint m^* \vec{v}_s \cdot d\vec{s} = n \frac{h}{2e} = n \Phi_0$$



$$B = 0$$

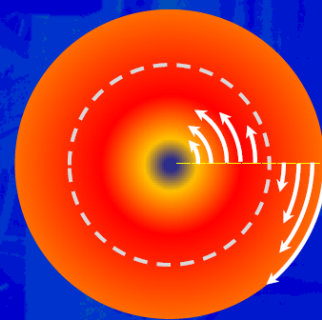
$$n = 0$$

$\odot \vec{B}$



$$B_1 > 0$$

$$n = 0$$



$$B_2 > B_1 > 0$$

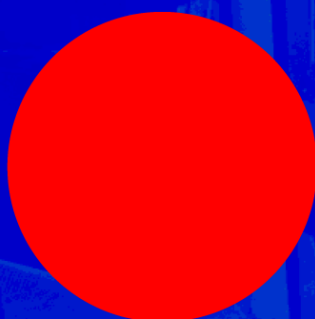
$$n = 1$$

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How should we understand STS data?

$$\vec{A} = \frac{1}{2} \vec{B} \wedge \vec{r}$$

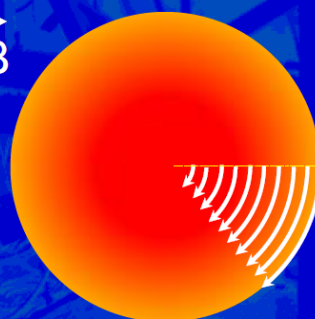
$$\Phi + \frac{1}{2e} \oint m^* \vec{v}_s \cdot d\vec{s} = n \frac{h}{2e} = n \Phi_0$$



$$B = 0$$

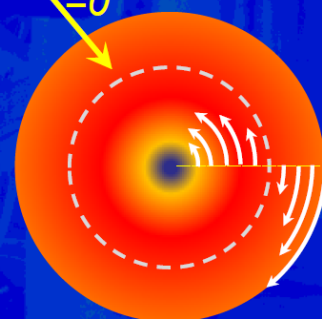
$$n = 0$$

$\odot \vec{B}$



$$B_1 > 0$$

$$n = 0$$

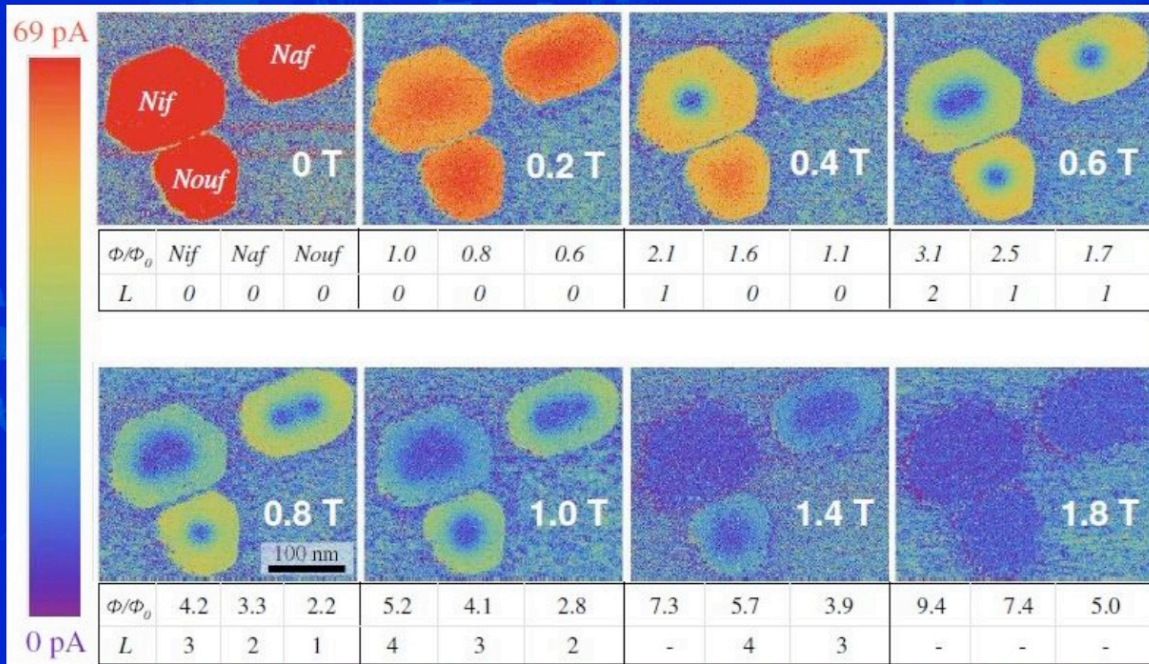


$$B_2 > B_1 > 0$$

$$n = 1$$

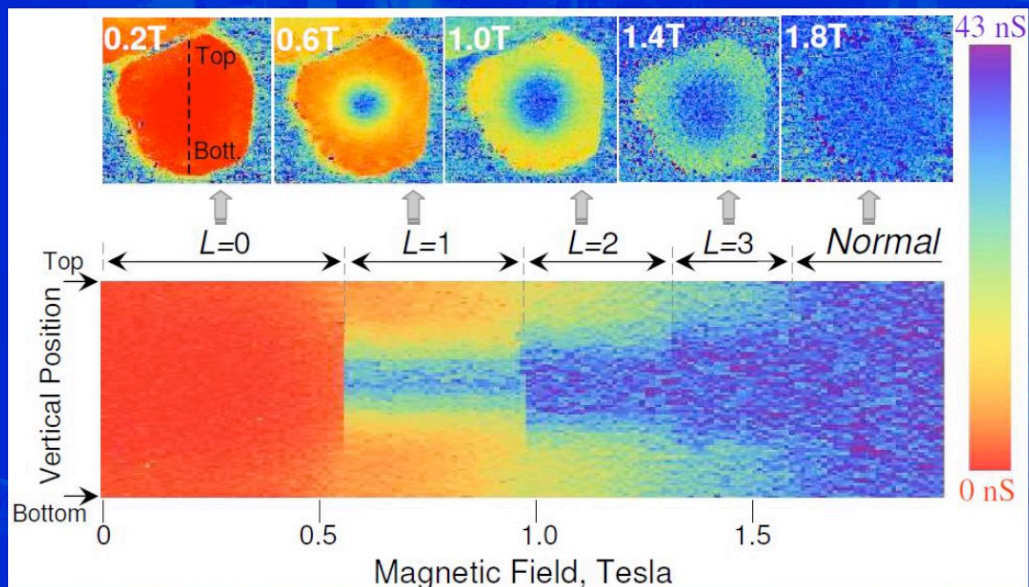
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Strongly Confined Superconducting Condensate



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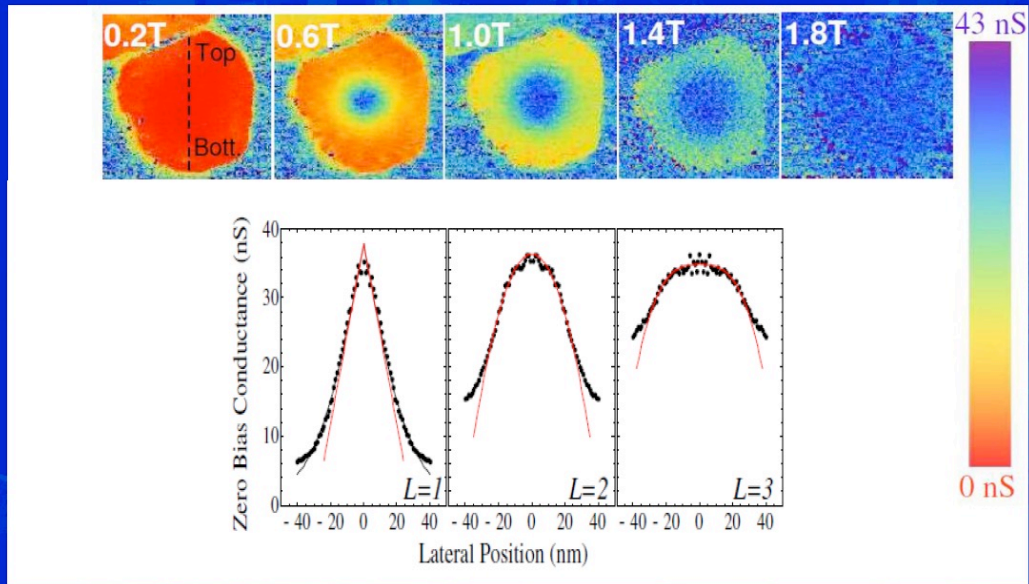
Response of Confined Superconducting Condensate to an External Magnetic Field: Giant Vortex States



T. Cren et al. PRL 107, 097202 (2011)

ISS2013, November 18-20, Tokyo

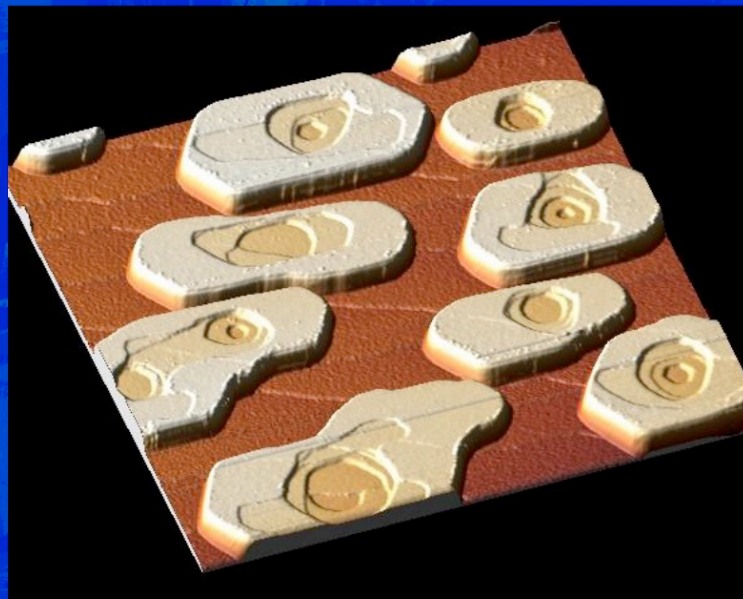
Response of Confined Superconducting Condensate to an External Magnetic Field: Giant Vortex States



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Boundary effects Pinning at the nanoscale

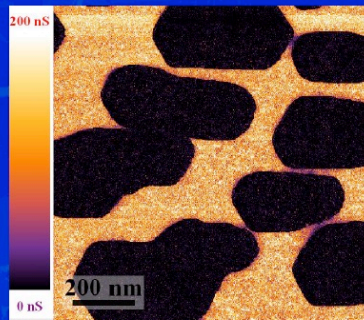
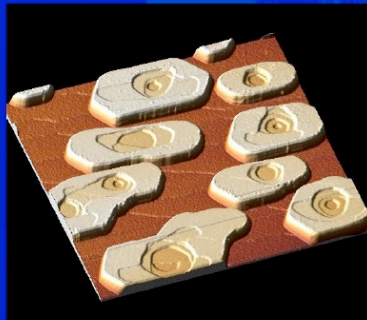


Pb/Si(111) $1 \times 1 \mu\text{m}^2$

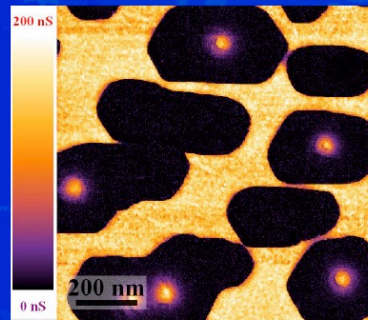
9-14 atomic layers thick superconducting islands

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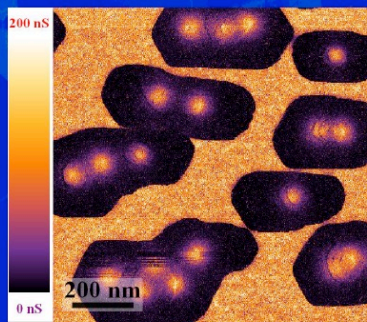
Boundary effects



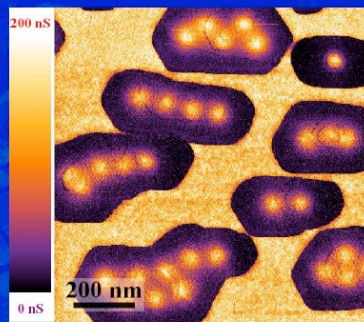
$T=300\text{mK}$, $B=0\text{mT}$



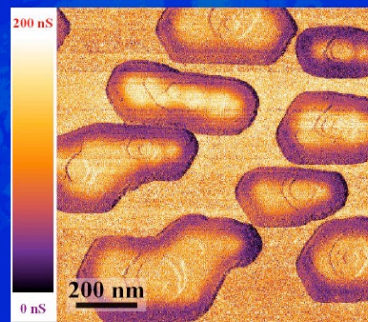
$B=60\text{mT}$



$B=120\text{mT}$



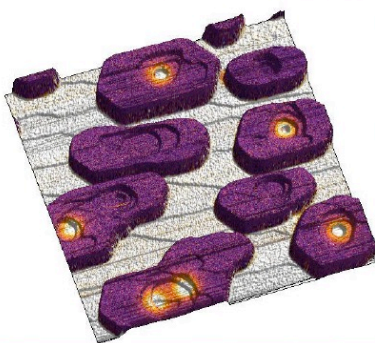
$B=180\text{mT}$



$B=360\text{mT}$

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Pinning at nanoscale



$B=60\text{mT}$



$B=120\text{mT}$



$B=180\text{mT}$

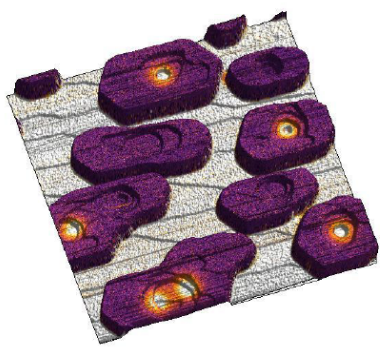
Pinning of the vortices in the holes

Deconfinement effect at high field

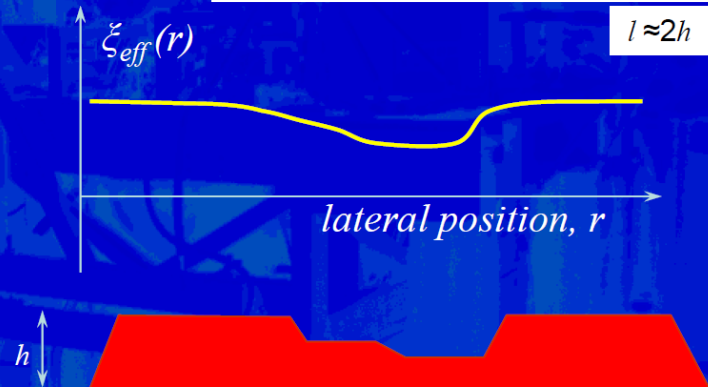
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Origin of pinning

$$\xi(T) = 0.855 \sqrt{\frac{\xi_0 l}{(1 - \frac{T}{T_{c0}})}} \quad (\xi_0 \gg l)$$



B=60mT

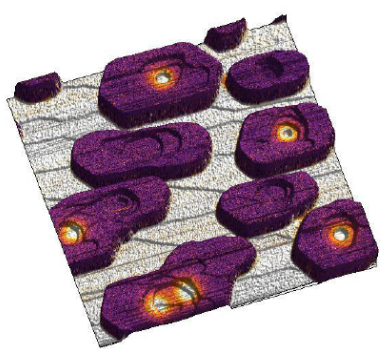


Vortex core energy E_{core}

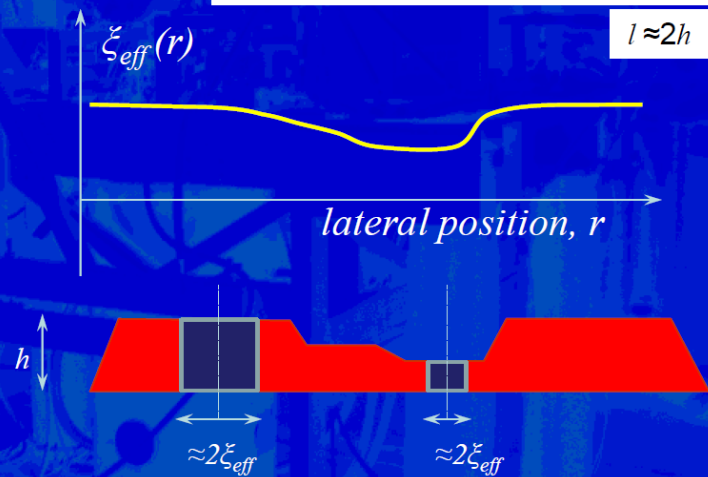
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Origin of pinning

$$\xi(T) = 0.855 \sqrt{\frac{\xi_0 l}{(1 - \frac{T}{T_{c0}})}} \quad (\xi_0 \gg l)$$



B=60mT



Vortex core energy $E_{core} \approx \Delta \pi \xi_{eff}^2 h \approx \Delta \pi \xi_0^2 2h h \sim h^2$

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Hour 10.

5.5. Magnetic field of a single Abrikosov vortex.

It is clear that the magnetic field crossing the normal state “hole” will be screened by supercurrents circulating around it on a typical scale λ . Let us calculate this field more precisely, considering the phase singularity $2\pi\delta(\vec{r})\vec{u}_z$ located at $(x, y) = 0$. From

$$\vec{J}_s = \frac{(2e)^2}{m^*} |\psi|^2 \left[\frac{\hbar}{2e} \vec{\nabla} \theta - \vec{A} \right]$$

and by using $\mu_0 \vec{J}_s = \text{rot}(\vec{B})$ we obtain:

$$\text{rot}(\vec{B}) = \frac{(2e)^2}{\mu_0 m^*} |\psi|^2 \left[\frac{\hbar}{2e} \vec{\nabla} \theta - \vec{A} \right]$$

Considering $r > \xi$, we have $|\psi|^2 \approx |\psi_0|^2 = \frac{|\tilde{a}(T_c - T)|}{b}$, and

$$\lambda(T) = \left(\frac{m^* b}{4\mu_0 e^2 |\tilde{a}(T_c - T)|} \right)^{1/2} = \left(\frac{m^*}{4\mu_0 e^2 |\psi|^2} \right)^{1/2}$$

we get, by taking $\text{rot}(\dots)$ from both parts of the equation,

$$\lambda^2 \text{rot}(\text{rot}(\vec{B})) + \vec{B} = \frac{\Phi_0}{2\pi} \text{rot}(\vec{\nabla} \theta)$$

or, by using the above defined delta-function:

$$\lambda^2 \text{rot}(\text{rot}(\vec{B})) + \vec{B} = \Phi_0 \delta(\vec{r}) \vec{u}_z$$

with the boundary condition $\vec{B}(\infty) = 0$. The solution of this equation is known, it is Macdonald function K_0

$$\vec{B}(r) = \frac{\Phi_0}{2\pi\lambda^2} K_0\left(\frac{r}{\lambda}\right)$$

$$K_0(x) = \begin{cases} \ln\left(\frac{1}{x}\right), & \text{for } x \ll 1 \\ \frac{e^{-x}}{\sqrt{x}}, & \text{for } x \gg 1 \end{cases}$$

This solution provides a rapid (almost exponential) screening of the field at large distances $r \gg \lambda$, but apparently diverges at $r \rightarrow 0$ as

$$\vec{B}(r) = \frac{\Phi_0}{2\pi\lambda^2} \ln(\lambda/r)$$

However, on the scale of ξ the OP vanishes, $|\psi|^2 \sim |r|^2 \rightarrow 0$. It means that close to the core, there are no more Cooper pairs and, therefore, no currents generating the field. We can introduce a cut-off at $r = \xi$ and obtain the field estimation at the singularity $r = 0$

$$\vec{B}(r = 0) = \frac{\Phi_0}{2\pi\lambda^2} \ln\left(\frac{\lambda}{\xi}\right) = \frac{\Phi_0}{2\pi\lambda^2} \ln(\kappa)$$

NB: Draw here the OP and B as the function of r.

5.6. First critical field.

Let us try to understand the region in the phase diagram where the vortex can exist. For this, **let us first consider the energy of a single vortex in a SC in zero-magnetic field**. This energy density is:

$$\varepsilon_{vortex} = \varepsilon_{core} + \varepsilon_{magn} + \varepsilon_{kin}$$

The core energy is usually neglected (see below). The magnetic energy density is

$$\varepsilon_{magn} = \frac{B^2}{2\mu_0}$$

It is positive since the SC state provides the energy necessary to create the field. The kinetic energy of moving Cooper pairs is

$$\varepsilon_{kin} = \frac{1}{2} n_s m^* v_s^2 = \frac{\lambda^2}{2\mu_0} \left(\text{rot}(\vec{B}) \right)^2$$

Putting the two terms together we get

$$E_{vortex} = \frac{1}{2\mu_0} \int dV \left[B^2 + \lambda^2 \left(\text{rot}(\vec{B}) \right)^2 \right]$$

Taking into account the cylindrical symmetry of the problem, we can calculate the vortex energy E_{vortex} per unit of length. To do this, we limit the integration over the volume limited in z -direction by two planes $z = 0$ and $z = 1$, and infinitely far from the vortex core in (x, y) -plane. We can use

$$\left(\text{rot}(\vec{B}) \right)^2 = (\vec{B}, \text{rot}(\text{rot}(\vec{B}))) - \text{div}(\text{rot}(\vec{B}) \times \vec{B})$$

We also see that $\int dV \text{div}(\text{rot}(\vec{B}), \vec{B}) = \oint d\vec{S} (\text{rot}(\vec{B}), \vec{B}) = 0$ (recall London), since $(\text{rot}(\vec{B}), \vec{B})$ lies in the $z = \text{cte}$ plane. The surface integral $d\vec{S}$ is taken over infinite surfaces $z = 0$ and $z = 1$ and over edge. At the infinite edge $\vec{B}(\infty) = 0$ and the integral is zero. At the surfaces $z = 0$ and $z = 1$ $d\vec{S}$ is perpendicular to $(\text{rot}(\vec{B}), \vec{B})$. Therefore we have

$$E_{vortex} = \frac{1}{2\mu_0} \int dV (\vec{B}, [\vec{B} + \lambda^2 \text{rot}(\text{rot}(\vec{B}))])$$

But $\vec{B} + \lambda^2 \text{rot}(\text{rot}(\vec{B})) = \Phi_0 \delta(\vec{r}) \vec{u}_z$. Thus the vortex energy per length unit is

$$E_{vortex} = \frac{\Phi_0}{2\mu_0} B(0) = \frac{\Phi_0^2}{4\pi\mu_0\lambda^2} \ln(\kappa)$$

Till now, we did not consider the energy of the normal core of the vortex per unit of length, which is of the order

$$E_{core} \approx \frac{B_c^2}{2\mu_0} \pi \xi^2$$

But we know that

$$B_c = \mu_0 H_c(T) = \frac{\Phi_0}{2\pi\sqrt{2} \lambda(T) \xi(T)}$$

We obtain

$$E_{core} \approx \frac{1}{4} \frac{\Phi_0^2}{4\pi\mu_0\lambda^2} \ll \frac{\Phi_0^2}{4\pi\mu_0\lambda^2} \ln(\kappa) = E_{vortex}$$

Nota. This is only true for superconductors with lateral dimensions $\gg \lambda$.

Thus, **the energy of a single vortex per unit of its length is positive**. Therefore in zero-field the vortex is not thermodynamically stable. However, at finite field the total Gibbs energy of a SC with a vortex inside could be lower than the Gibbs energy without the vortex:

$$\begin{aligned} G_s^{novortex} &= G_s(H=0) - \mu_0 \int dV \vec{M} \vec{H}_{ext} = G_s(0) - \int dV \vec{B} \vec{H}_{ext} + \mu_0 \int dV \vec{H}_{ext} \vec{H}_{ext} \\ &= G_s(0) + \mu_0 \frac{H_{ext}^2}{2} \cdot S \end{aligned}$$

$$\begin{aligned} G_s^{vortex} &= G_s(H_{ext}=0) + E_{vortex} - \mu_0 \int dV \vec{M} \vec{H}_{ext} \\ &= G_s(0) + E_{vortex} - \int dV \vec{B} \vec{H}_{ext} + \mu_0 \frac{H_{ext}^2}{2} \cdot S \end{aligned}$$

Plot the branches.

The term E_{vortex} increases the energy but now the term $-\int dV \vec{B} \vec{H}$ is non-zero, and it could be negative. In fact, the phase accumulation could be 2π or -2π , which decides in which direction the vortex currents circulate. If their direction is such that they generate locally the field $\vec{B}$ in the same direction as $\vec{H}$, then, the integral is negative,

$$-\int dV \vec{B} \vec{H}_{ext} = -\vec{H}_{ext} \int d\vec{S} \vec{B} = -\vec{H}_{ext} \Phi_0$$

Therefore vortex phase becomes energetically favorable when

$$\vec{H}_{ext} \Phi_0 = E_{vortex}$$

We thus obtain the formula for the so-called first critical field:

$$\mu_0 H_{c1} = \mu_0 \frac{E_{vortex}}{\Phi_0} = \frac{\Phi_0}{4\pi\lambda^2} \ln(\kappa)$$

This field is smaller than the “thermodynamical” critical field H_C by a factor

$$\frac{H_{c1}}{H_C} \approx \frac{\xi}{\lambda} \ln(\kappa) < 1$$

Since we know the temperature dependence for $\lambda(T)$ we get:

$$H_{c1}(T \rightarrow T_c) = H_{c1}(0)(T_c - T)$$

5.7. Second critical field.

At $H > H_{c1}$ as the external field raises, more and more vortices appear. The vortices interact magnetically and by currents (Bernoulli) and repel each other. They settle in the so-called Abrikosov vortex lattice.

Nota. The transitions between branches characterized by a given number n of vortices are topological transitions: the total number of 2π phase winds in the sample is n ; therefore, the transition from n to $n \pm 1$ is not continuous. Moreover, there exists a surface barrier (*explain*) that makes these transitions hysteretic (*draw*).

NB: Show here PowerPoint with the vortex lattice in SC but also in BEC and superfluid He. End with the possibility to study confinement effects.

When the distance between AV becomes smaller than λ the external magnetic field penetrates everywhere in the sample; the latter remains in the zero-resistivity state since the OP is non-zero everywhere except vortex cores $\sim \xi$. It is easy to see that in this case, the inter-vortex distance a is inversely proportional to the square root of the magnetic field. Indeed, since by topology, the edge of each vortex lattice unit cell contains no current, the total flux over one unit cell is exactly Φ_0 . Thus we have:

$$\mu_0 H \approx B = \Phi_0 / S_{uc} = \Phi_0 / \frac{\sqrt{3}}{2} a^2$$

We thus get

$$a = \sqrt{\frac{2\Phi_0}{\sqrt{3}\mu_0 H}}$$

NB. At 1T the inter-vortex distance is 50nm (if we neglect the field screening, i.e. if $\lambda \gg 50nm$)).

When this distance a becomes close to the normal core size, typically $a \approx 2\xi$, the vortex cores overlap and the SC state is destroyed everywhere. The corresponding critical field is called “second critical field” is:

$$\mu_0 H_{C2} = B_{C2} = \frac{\Phi_0}{2\pi\xi^2}$$

That is higher than the thermodynamic critical field $H_C = \frac{H_{C2}}{\sqrt{2}\kappa}$.

Nota. We can now compare the three critical fields, namely

$$B_c = \mu_0 H_C(T) = \frac{\Phi_0}{2\pi\sqrt{2}\lambda\xi}$$

$$B_{C1} = \mu_0 H_{C1} = \mu_0 \frac{E_{vortex}}{\Phi_0} = \frac{\Phi_0}{4\pi\lambda^2} \ln(\kappa)$$

, and B_{C2} . We get, for strong type II superconductors, $\kappa = \lambda/\xi \gg 1$,

$$\frac{\ln(\kappa)}{\sqrt{2}\kappa} B_c = B_{C1} < B_c \ll B_{C2} = \sqrt{2}\kappa B_c$$

This means that owing to the vortex phase, the zero-resistance state is preserved up to fields B_{C2} much higher than the thermodynamical critical field B_c , even if the Meissner state exists only at low fields, $\mu_0 H_{ext} < B_{C1}$.

5.8. Third critical field.

There is a very interesting effect: while vortices “kill” the SC state at H_{C2} , near the sample surface (edge) in a very thin layer $\sim \xi$ there is specific compensation of vortex currents by Meissner currents. As a result, the OP does not vanish there at H_{C2} but at a slightly higher field

$$H_{C3} \approx 1.69H_{C2}$$

N.B. Vortex phase – possibility of energy dissipation (vortex core is normal !!)/ In general – vortex motion = dissipation. Discuss this a bit and show PowerPoint about applications and vortex confinement.