

Superconductivity - M2

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TD 4: Josephson physics

1 Feynman's derivation of the DC Josephson effect

In this exercise, we want to derive the equation for the DC Josephson effect in a different manner than in the lecture. For this, we consider two superconductors made out of the same material, separated by an insulating barrier (SIS junction). Both are then connected to a battery, so that there is a potential difference V across the junction. Let us consider the situation of zero magnetic field.

We call ψ_A the amplitude to find an electron on side A , and ψ_B the amplitude to find it on side B . In the superconducting state, ψ_i is the common wavefunction of all electrons of side i . The two amplitudes are related in the following way:

$$\begin{aligned} i\hbar \frac{\partial \psi_A}{\partial t} &= U_A \psi_A + K \psi_B \\ i\hbar \frac{\partial \psi_B}{\partial t} &= K \psi_A + U_B \psi_B \end{aligned}$$

1. What is the physical meaning of U_i and K . What is the relation between U_A and U_B ? In the following, we will take $U_A = eV/2$ and $U_B = -eV/2$.
2. Introduce $\psi_i = \sqrt{n_i} e^{i\theta_i}$, with n_i the superfluid density of side i and θ_i its phase, and derive the equation for the DC Josephson effect.
3. Derive the equation for the AC Josephson effect.

2 DC SQUIDS

Superconducting Quantum Interference Devices (SQUIDS) consist in 2 Josephson junctions ("a" and "b") in parallel, as shown in figure 1. We consider the situation where the magnetic field perpendicular to the plane of the SQUID loop may be non-zero. The two arms of the SQUID are symmetrical (i.e. of same length) and are assumed to be thicker than λ_L .

Let δ_a be the phase difference between both superconductors through junction a , δ_b the one between both superconductors through junction b , and δ_0 the one between both superconductors in the absence of magnetic field.

1. Determine the phase differences δ_a and δ_b as a function of δ_0 , ϕ_0 the flux quantum, and ϕ the magnetic flux across the junction.
2. Determine the total current I_{tot} that flows from point P to point Q .
3. Explain why SQUIDS are extremely sensitive magnetometers.

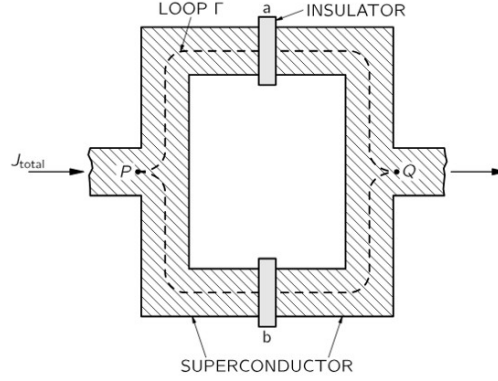


Figure 1: Schematic of a SQUID. Figure taken from *The Feynman Lectures on Physics* (vol. 3, ch. 21).

Exercise: RF-driven Josephson junction

When a Josephson junction is irradiated with an electromagnetic wave of angular frequency ω_1 , so-called “Shapiro steps” are observed in the dc $I - V$ curve at voltages. This is what we will explain in this exercise.

The Bessel functions of the first kind are given by the Jacobi–Anger identity

$$e^{ix \sin \theta} = \sum_{n=-\infty}^{\infty} J_n(x) e^{in\theta}, \quad J_{-n}(x) = (-1)^n J_n(x).$$

1. First, let us consider the case where a constant DC voltage V_0 is applied across the Josephson junction. Write $\gamma(t)$ and $I_s(t)$. What is the time-averaged supercurrent? Give the frequency f_J of the AC Josephson current for $V_0 = 1 \mu\text{V}$.
2. The junction is now biased with $V(t) = V_0 + V_1 \cos(\omega_1 t)$. Show that

$$\gamma(t) = \gamma_0 + \omega_0 t + a \sin(\omega_1 t).$$

Give ω_0 and a in terms of V_0 , V_1 and ω_1 . What is the physical meaning of γ_0 ?

3. Use the Jacobi–Anger identity to show that

$$I_s(t) = I_c \sum_{n=-\infty}^{\infty} J_n(a) \sin(\gamma_0 + (\omega_0 + n\omega_1)t).$$

4. Compute the time average $\langle I_s \rangle$. Show that it vanishes unless V_0 takes one of the discrete values

$$V_n = n \frac{\hbar \omega_1}{2e}, \quad n \in \mathbb{Z}.$$

Give the expression for $\langle I_s \rangle$ at $V_0 = V_n$.

5. At $V_0 = V_n$, γ_0 is not fixed by the voltage source. Deduce the height ΔI_n of the n -th step.
6. What is the expected $I(V)$ curve for $f_1 = \omega_1/2\pi = 10 \text{ GHz}$?
7. In real life, Josephson junctions are usually current biased. Without any calculation, sketch the resulting $V(I)$.