Examen 8/11/2025 - Méca.

Exercice 1. /2

1) Faux : peut être en mouvement rectiligne uniforme.
2) Vrai.
3) Faux : si $\vec{u} \parallel \vec{v}$ $\vec{u}\cdot\vec{v} = \|\vec{u}\|\|\vec{v}\|$.
4) Faux : si $\vec{F} \perp$ trajectoire, le travail est nul.
5) Vrai.
6) Vrai.
7) Vrai.
8) Faux : $\vec{\mathcal{M}}_{\vec{F}} = \vec{r} \wedge \vec{F} \perp \vec{F}$.

Exercice 2. /1.5

1) Indirecte 2) Directe.

3) $= 4\vec{u_z} - 6\vec{u_y} - \vec{u_z} - 3\vec{u_x} - 5\vec{u_y} - 10\vec{u_x} = -13\vec{u_x} - 11\vec{u_y} + 3\vec{u_z}$.

4) $= -2\vec{u_z} - 8\vec{u_\theta} - 8\vec{u_r}$.

5) $= -8\vec{u_x}$ 6) $= -5\vec{u_z}$.

Exercice 3. /2.25

.5 1) $\vec{u_r} = \cos\theta\,\vec{u_x} + \sin\theta\,\vec{u_y}$ $\vec{u_\theta} = -\sin\theta\,\vec{u_x} + \cos\theta\,\vec{u_y}$.

0.5 2) $\vec{OM} = R\cos\theta\,\vec{u_x} + R\sin\theta\,\vec{u_y} = R\vec{u_r}$.

0.5 3) $\vec{v} = -R\dot\theta\sin\theta\,\vec{u_x} + R\dot\theta\cos\theta\,\vec{u_y} = R\dot\theta\,\vec{u_\theta}$.

0.5 4) $\vec{a} = -R\ddot\theta\sin\theta\,\vec{u_x} - R\dot\theta^2\cos\theta\,\vec{u_x} + R\ddot\theta\cos\theta\,\vec{u_y} - R\dot\theta^2\sin\theta\,\vec{u_y}$.

$\vec{a} = +R\ddot\theta\,\vec{u_\theta} - R\dot\theta^2\,\vec{u_r}$.

0.25 5) $\vec{\omega} = \dot\theta\,\vec{u_z}$.

Exercice 4. /4.5

0.25 1) $v(t) = R(t)\,\omega(t)$ 0.25 2) $I = mR^2$ 0.25 3) $\vec{L} = mR^2\omega\,\vec{u_z}$.

0.25 4) $\dfrac{d\vec{L}}{dt} = \Sigma \vec{\mathcal{M}}_{F_{ext}}$ 0.25 5) ⓐ $\vec{\mathcal{M}}_{\vec{F}} = \vec{0}$ 0.25 ⓑ $\vec{L} = \vec{cste} = mR_0^2\omega_0\vec{u_z} = mR_0v_0\vec{u_z}$

0.25 ⓒ A tout instant $R(t)v(t) = R_0v_0$ car $\vec{\mathcal{M}}_{F_{ext}} = \vec{0}$. ⓓ Il doit diminuer R. 0.25

ⓔ $v = 10\ m.s^{-1}$.
0.25

0.25 6) ⓐ $\lambda = \frac{m}{2\pi R}$ $\quad I = \int_0^{2\pi} \lambda R^2 R d\theta = 2\pi\lambda R^3 = mR^2$ 0.5

0.5 ⓑ $\frac{d\vec{L}}{dt} = \frac{d}{dt}(mR^2\omega) = 2mR\omega + mR^2\frac{d\omega}{dt} = \vec{\Gamma}_{\vec{P}} + \vec{\Gamma}_{\vec{T}}$

0.25 ⓒ $\vec{\Gamma}_{\vec{P}} = \vec{0}$ par symmétrie ⇒ ne contribue pas.

0.25 ⓓ $\lambda R d\theta (R\ddot{\theta}\vec{u_\theta} - R\dot{\theta}^2\vec{u_r}) = \Sigma\vec{F}$.

Projection sur $\vec{u_r}$: $-\lambda R^2\dot{\theta}^2 d\theta \vec{u_r} = -2T\sin\frac{d\theta}{2}\vec{u_r} = -T d\theta\vec{u_r}$.

0.25 D'où $T = \lambda R^2\omega^2$

0.25 ⓔ $T \nearrow, R \searrow \Rightarrow \omega \nearrow$ ça tourne plus vite.

Exercice 5. /2

1) $\vec{a} = \begin{vmatrix} 0 \\ 0 \\ -g \end{vmatrix}$ $\quad \vec{v} = \begin{vmatrix} 0 \\ v_0\cos\alpha \\ -gt + v_0\sin\alpha \end{vmatrix}$ $\quad \vec{OG} = \begin{vmatrix} 0 \\ v_0 t\cos\alpha \\ -\frac{1}{2}gt^2 + v_0 t\sin\alpha + H_0 \end{vmatrix}$.

0.5

2) $y(t) = v_0 t\cos\alpha \Rightarrow t = \frac{y}{v_0\cos\alpha}$

0.5 $z(t) = -\frac{1}{2}g\frac{y^2}{v_0^2\cos^2\alpha} + y\,tg\,\alpha + H_0$.

3) Si $y = D_{AV}$ $z = 1.9$ m pour la trajectoire ⇒ non.

Si $y = D_{Ja}$ $z = 1.59$ m " " ⇒ non.

0.5 Si $y = D_{Wi}$ $z = 1.23$ m " " ⇒ Lucky Luke attrape William.

Si $y = D_{Jo}$ $z = 0.44$ m " " ⇒ non.

4) On veut : $(-H_{Jo} + H_0 + D_{Jo}\,tg\,\alpha) \times 2 = g\frac{D_{Jo}^2}{v_0^2\cos^2\alpha}$

0.5 $v_0 = 9.91\ m.s^{-1}$.

Exercice 6. /3.25

0.25 1) Oz $\quad$ 0.25 2) $\vec{OM_i} = OM_i\cos\theta\,\vec{u_x} + OM_i\sin\theta\,\vec{u_z}$. $\quad$ $OM_i > 0$ pour M_3, M_4; $OM_i < 0$ pour M_1, M_2

3) $\vec{\Gamma_1} = +\overset{\cos\theta_1}{\vee}OM_1\,\vec{u_x}\wedge m_1 g\,\vec{u_z} = -m_{Jo}g\,OM_1\,\vec{u_y}$.

$\vec{\Gamma_2} = -m_{Ja}g\,OM_2\,\vec{u_y}\cos\theta$ $\quad$ $\vec{\Gamma}_{axe} = \vec{0}$

1.5 $\vec{\Gamma_3} = +m_{Wi}g\,OM_3\,\vec{u_y}\cos\theta$ $\quad$ $\vec{\Gamma}_{poids\ pomme} = \vec{0}$

$\vec{\Gamma_4} = +m_{Av}g\,OM_4\,\vec{u_y}\cos\theta$

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0.5 4) $\Gamma_1 + \Gamma_2 = 89\,\overset{r\cos\theta g}{kg}\,m^2 s^{-2}$ et $\Gamma_3 + \Gamma_4 = 272.5\,\overset{r\cos\theta g}{kg}.\,m^2.s^{-2}$ => vers Averell.

0.5 5) Par exemple Jack à 10 cm et Averell à 80 cm de O.

Exercice 7.

/5

0.5 1) $\vec{F} = -\frac{\eta_0 m G}{R^2}\vec{u_r}$ $\quad E_p = -\frac{\eta_0 m G}{R}$.

0.25 2) (a) $\vec{a} = R\ddot{\theta}\,\vec{u_\theta} - R\dot{\theta}^2\vec{u_r} = -\frac{\eta_0 G}{R^2}\vec{u_r} \Rightarrow \ddot{\theta} = 0 \Rightarrow \dot{\theta} = \text{cste}$.

0.25 (b) On a $R\dot{\theta}^2 = \frac{\eta_0 G}{R^2} = \frac{v^2}{R} \Rightarrow v = \sqrt{\frac{\eta_0 G}{R}}$.

0.25 (c) $E_m = -\frac{\eta_0 m G}{R} + \frac{1}{2}m\frac{\eta_0 G}{R} = -\frac{1}{2}\frac{m\eta_0 G}{R} = E_m$.

0.25 (d) $W_{\vec{F}} = 0$ car $\vec{F} \perp$ trajectoire.

0.25 (e) $\Delta E_m = \sum W_{F_{\text{non cons}}} = 0. \Rightarrow$ Em ne varie pas.

0.25 3) $\vec{\Gamma}_{\vec{F}} = \vec{0}$.

0.25 4) $\vec{L} = mR\vec{u_r} \wedge \sqrt{\frac{\eta_0 G}{R}}\,\vec{u_\theta} = \sqrt{m^2\eta_0 G R}\;\vec{k}$.

0.25 5) $\frac{d\vec{L}}{dt} = \sum \vec{\Gamma}_{\vec{F}} = \vec{0} \Rightarrow \vec{L} = \vec{\text{cste}} \Rightarrow$ trajectoire plane.

0.5 6) 

L'aire parcourue pdt dt : $d\mathcal{A} = \frac{1}{2}Rv\,dt = \frac{L}{2m}dt$.

D'où $\pi R^2 = \frac{L}{2m}T = \frac{1}{2}\sqrt{\eta_0 G R}\;T$

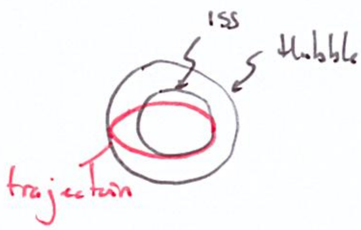
$$4\pi^2 R^4 = \eta_0 G R T^2$$

$$\frac{T^2}{R^3} = \frac{4\pi^2}{\eta_0 G}$$

0.75 7) $\frac{T_S^2}{R_S^3} = \frac{T_H^2}{R_H^3} \Rightarrow T_S = T_H\left(\frac{R_S}{R_H}\right)^{3/2} = 92.9$ min.

$V_H = \frac{2\pi R_H}{T_H} = 7548\ m.s^{-1}$.

$V_S = \frac{2\pi R_S}{T_S} = 7661\ m.s^{-1}$.



0.75 8) (a) $\frac{1}{2}\cancel{m}v^2 = -\frac{\cancel{m}\eta_0 G}{R_S + R_H} + \frac{\cancel{m}\eta_0 G}{R_H}$

$$v^2 = \eta_0 G\frac{(-R_H + R_S + R_H)}{R_H(R_S + R_H)}$$

$V_A = \sqrt{\frac{\eta_0 G R_S}{R_H(R_S + R_H)}}$ $\qquad V_P = \sqrt{\frac{\eta_0 G R_H}{R_S(R_S + R_H)}}$

=> utiliser des moteurs-fusée.

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0.5 (b) Durée $= \frac{1}{2} T = \sqrt{\frac{4\pi^2 R^3}{M_o G}} \times \frac{1}{2} = 47.4$ min.

Exercice 8.

/2.25

0.25 1) $\lambda = \frac{m}{L}$

0.5 2) $I = \int_{l=0}^{L} \frac{m}{L} l^2 \, dl = \frac{1}{3} m \frac{L^3}{L} = \frac{1}{3} m L^2.$

1 3) $\vec{L} = \frac{1}{3} m L^2 \dot{\theta} \, \vec{u_y}.$

$$\frac{d\vec{L}}{dt} = \vec{r} \wedge - mg \vec{u_z} = -\frac{L}{2} mg \left(\cos\theta \vec{u_z} + \sin\theta \vec{u_x}\right) \wedge \vec{u_z}.$$

$$= +\frac{L}{2} mg \sin\theta \, \vec{u_y}.$$

$$= \frac{1}{3} m L^2 \ddot{\theta} \, \vec{u_y}.$$

D'où : $\frac{1}{2} g \sin\theta = \frac{1}{3} L \ddot{\theta}$

$$L\ddot{\theta} = \frac{3}{2} g \sin\theta$$

$$L\dot{\theta}\ddot{\theta} = \frac{3}{2} g \dot{\theta} \sin\theta.$$

$$L \frac{1}{2} \dot{\theta}^2 = \frac{3}{2} g (-\cos\theta + \cos\theta_0)$$

$$\dot{\theta} = \sqrt{\frac{3g}{L} (\cos\theta_0 - \cos\theta)} = \frac{d\theta}{dt}$$

0.5 4) $t = \int_{\theta=\theta_0}^{\pi/2} \frac{d\theta}{\sqrt{\cos\theta_0 - \cos\theta}} \frac{L}{3g} = 5.1 \frac{L}{3g} = 5.2\text{s}.$