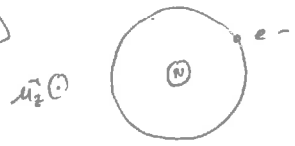
TD n°3.

Exercice 2 - Moment cinétique.

1)



$\vec{u_z}$ ⊙

$\vec{L_O} = m \vec{r} \wedge \vec{v} = m r^2 \dot{\theta} \vec{u_z} = \underline{1.06 \cdot 10^{-34} \vec{u_z}}$ [kg m² s⁻¹].

$f = 6.6 \cdot 10^{15}$ Hz $\Rightarrow$ Parcourt 2π en $1.51 \cdot 10^{-16}$ s.

$\Rightarrow \omega = 4.14 \cdot 10^{6}$ rad.s⁻¹ $= \dot{\theta}$

2)

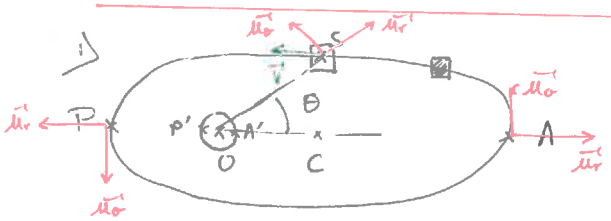
$\vec{u_z}$ ⊙

$\vec{L_O} = m r^2 \omega \vec{u_z} = m r^2 \frac{2\pi}{T} \vec{u_z} = \underline{2.81 \cdot 10^{38} \vec{u_z}}$ [kg m² s⁻¹]

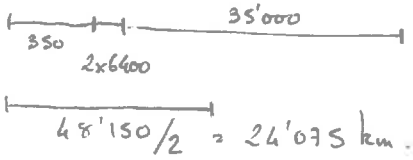
⚠ r en m.

Exercice 3 - Vitesse d'un satellite sur son orbite elliptique.

1)



⊙ $\vec{u_z}$



48'150/2 = 24'075 km.

2) $\vec{L_O} = m \vec{r} \wedge \vec{v} = m r \vec{u_r} \wedge (-v_s \cos\theta \vec{u_r} + v \sin\theta \vec{u_\theta})$.

$= m r v_s \sin\theta \vec{u_z}$.

$= m r v_s \frac{SC}{\sqrt{OC^2 + SC^2}} = \underline{6.08 \cdot 10^{13} \vec{u_z}}$ [kg m² s⁻¹].

Or $OC = OA' + A'C = OA' + A'A - AC = 17'325$ km.

$\Rightarrow OS = 24'074$ km.

et $v_s = 14650$ km/h $= 4069$ m/s.

3) Conservation du mt cinétique :

En P $\quad \vec{L_O} = m \, OP \, \vec{u_r} \wedge v \vec{u_\theta} \Rightarrow v = \frac{\|\vec{L_O}\|}{m \, OP} = 6'996$ m/s $= \underline{25'185}$ km/h.

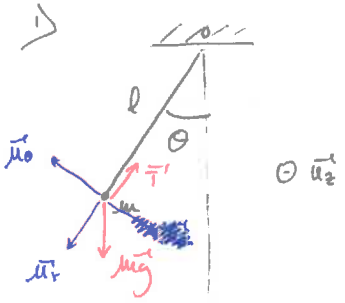
(OP = 6750 km)

En A $\quad v = \frac{\|\vec{L_O}\|}{m \, OA} = 1'135$ m/s $= \underline{4'087}$ km/h.

(OA = 41'400 km)

Exercice 4 : Pendule simple

1)



2) $\overrightarrow{OM} = r\vec{u}_r$

3) $\vec{\Gamma}_{mg,O} = \vec{r} \wedge \vec{F} = r\vec{u}_r \wedge \left(mg\cos\theta\, \vec{u}_r - mg\sin\theta\, \vec{u}_\theta \right)$.

$= -rmg\sin\theta\, \vec{u}_z \underline{\sim -rmg\theta\, \vec{u}_z}$.

4) $\vec{L} = m\vec{r} \wedge \vec{v} = \underline{mr^2\omega\, \vec{u}_z}$.

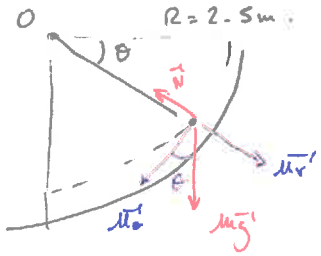
5) $\frac{d\vec{L}}{dt} = \vec{\Gamma}'_{mg,O} \Rightarrow -rmg\theta = mr^2\ddot{\theta} + 2mr\dot{\theta}\dot{r}$ (terme $\dot{r} = 0$).

$\underline{\ddot{\theta} = -\frac{g}{l}\theta} \Rightarrow \ddot{\theta} + \frac{g}{l}\theta = 0$

$\underline{\theta = \theta_0 \cos\sqrt{\frac{g}{l}}t}$

Exercice 5 : Vitesse d'un enfant à la sortie d'un toboggan

1)



2) $\frac{d\vec{L}_O}{dt} = \vec{\Gamma}'_{mg,O}$

$\vec{L}_O = mr\vec{u}_r \wedge r\dot{\theta}\vec{u}_\theta = mr^2\dot{\theta}\vec{u}_z$.

$\vec{\Gamma}'_{mg,O} = \vec{r} \wedge \vec{F} = r\vec{u}_r \wedge mg\cos\theta\, \vec{u}_\theta = mgr\cos\theta\, \vec{u}_z$.

$mr^2\ddot{\theta} = mgr\cos\theta$

$\underline{\ddot{\theta} = \frac{g}{OA}\cos\theta}$

3) $\dot{\theta}\ddot{\theta} = \frac{g}{OA}\dot{\theta}\cos\theta$

$\frac{d\dot{\theta}^2}{dt} \times \frac{1}{2} = \frac{g}{OA}\frac{d}{dt}(\sin\theta) \Rightarrow \dot{\theta}^2 = \frac{2g}{OA}\sin\theta + cste$.

à $t=0$ $\quad 0 = \frac{2g}{OA}\sin\theta_0 + cste$

$\underline{\dot{\theta}^2 = \frac{2g}{OA}(\sin\theta - \sin\theta_0)}$.

$\underline{v = r\dot{\theta} = \sqrt{2gr(\sin\theta - \sin\theta_0)}}$.

4) Vitesse max en $\theta = \pi/2$

$\underline{v = \sqrt{2gr(1 - \sin\theta_0)} = 6\ m/s}$

ce que l'on retrouve via conservation de l'énergie.