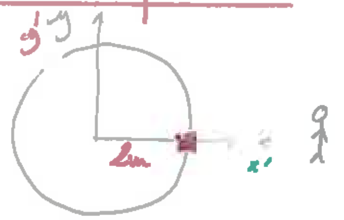
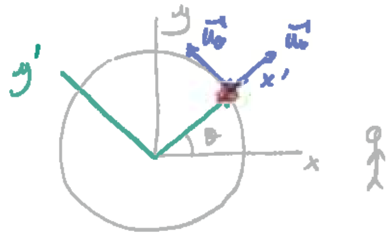
TD n° 1

Ex1 - Coord. polaires

1)





- Traj. circul^{re}
- $x = R\cos\theta$ $\quad R^* = \sqrt{x^2+y^2}$

  $y = R\sin\theta$

2) (R,θ) = coord. polaires

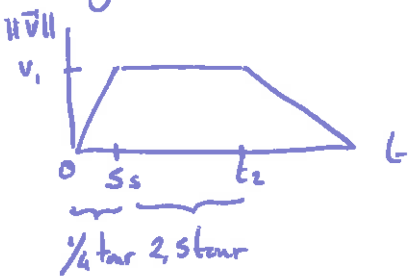
$$\begin{cases} R = \sqrt{x^2+y^2} \\ \theta = \text{Arctg}\,\dfrac{y}{x} \end{cases}$$

3) 1/4 tour $\quad L = \dfrac{\pi R}{2} = 3.14\ m.$

1 tour $\quad L = 2\pi R = 12,6\ m$

angle θ $\quad L = \theta R . = 2\theta\ [m]$

4)



ⓐ $s(t) = \displaystyle\int_{t=0}^{5s} kt\,dt = \frac{1}{2}kt^2 = \frac{25}{2}k.$

$= \dfrac{\pi R}{2}$

$\Rightarrow k = \dfrac{\pi}{25}R = 0.25\ m.s^{-2}$

ⓑ $v_1 = k \times 5s = 1.25\ m.s^{-1}.$

$v_1\,(t_2 - 5) = 2,5 \times 2\pi \times R$

$t_2 = \dfrac{2,5 \times 2\pi \times R}{1,25} + 5 = 30s = t_2.$

ⓒ $v = v_1 - k(t_3 - t_2) = 0$

$\dfrac{v_1}{k} + t_2 = t_3 = 35s$

5) ⓐ $\vec{OM} = R\,\vec{u_r}.$

$\vec{u_r} = \cos\theta\,\vec{u_x} + \sin\theta\,\vec{u_y}$

$\vec{u_\theta} = -\sin\theta\,\vec{u_x} + \cos\theta\,\vec{u_y}$

ⓑ $\dfrac{d\vec{u_r}}{dt} = \dot{\theta}\left(-\sin\theta\,\vec{u_x} + \cos\theta\,\vec{u_y}\right) = \dot{\theta}\,\vec{u_\theta}$

$\vec{v} = R\dot{\theta}\,\vec{u_\theta}.$

ⓒ $\vec{a} = R\ddot{\theta}\,\vec{u_\theta} + R\dot{\theta}^2 \underbrace{\left(-\cos\theta\,\vec{u_x} - \sin\theta\,\vec{u_y}\right)}_{-\vec{u_r}} = \underbrace{R\ddot{\theta}\,\vec{u_\theta} - R\dot{\theta}^2\,\vec{u_r}}_{-\frac{v^2}{R}\vec{u_r}} = \vec{a}$

ⓓ $[0, t_1]$

$R\dot{\theta} = kt \Rightarrow \vec{v} = kt\,\vec{u_\theta}$

$$\left.\begin{array}{l} R\ddot{\theta} = k \\ -\frac{v^2}{R} = -\frac{k^2t^2}{R} \end{array}\right\} \Rightarrow \vec{a} = k\,\vec{u_\theta} - \frac{k^2t^2}{R}\,\vec{u_r}$$

$[t_1, t_2]$

$\vec{v} = v_1\,\vec{u_\theta}$

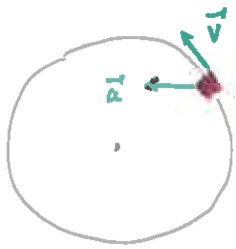
$\vec{a} = \vec{0} - \frac{v_1^2}{R}\,\vec{u_r}$

$[t_2, t_3]$

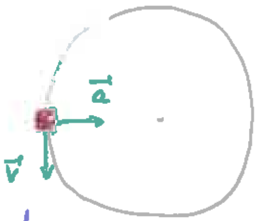
$\|\vec{v}\| = v_1 - k(t-t_2) \Rightarrow \vec{v} = \left(v_1 - k(t-t_2)\right)\vec{u_\theta}$

$$\left.\begin{array}{l} R\ddot{\theta} = -k \\ -\frac{v^2}{R} = (\quad)^2 \end{array}\right\} \Rightarrow \vec{a} = -k\,\vec{u_\theta} - \frac{[v_1 - k(t-t_2)]^2}{R}\,\vec{u_r}$$

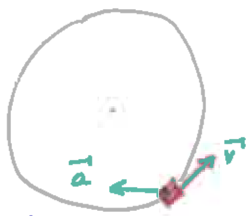
ⓔ



$\frac{1}{8}$ tour $\Rightarrow \in [0, t_1]$

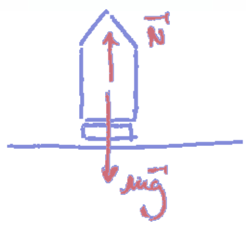


$\frac{1}{2}$ tour $\Rightarrow \in [t_1, t_2]$



2 tours $\frac{7}{8} \Rightarrow \in [t_2, t_3]$

Ex 5 - Lancement d'une fusée.

1) 

Syst. pseudo-isolé : $\sum \vec F = \vec 0$.
↳ réf inertiel : CdM immobile.
$\Rightarrow \frac{d\vec p}{dt} = \vec 0$.

Qd gaz st éjectés 

Au mt de l'éject°, CdM immobile
$\Rightarrow$ fusée poussée.

Ds l'espace $\rightarrow$ syst. isolé $\Rightarrow \frac{d\vec p}{dt} = \vec 0$.

$\Delta \vec p = \vec 0 = (m+M)\vec v = -m\vec u + M\vec v' \Rightarrow \vec v' = \frac{(m+M)\vec v + m\vec u}{M}$

$\vec v' = \vec v + \frac{m}{M}(\vec v + \vec u)$

2) Combust° éjecte les gaz $\Rightarrow$ f. s/ le sol $\Rightarrow$ le sol exerce une f. opposée. $\Rightarrow$ dpd du débit d'éject°.

3) 

Eject° des gaz : $M_{tot}(t) = M_0 + m_0 - at$

$M(t)\frac{d\vec v}{dt} = -M(t)g + \underbrace{au}_{\text{f. de poussée}}$

Pr qu'il y ait décollage il faut $\left\|\frac{d\vec v}{dt}\right\| > 0$

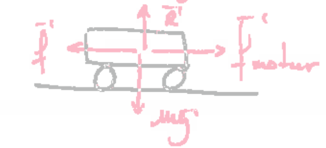
$M(t=0)\, g < au$.

A.N. $au = 10^6 \text{ kg m s}^{-2} = 10^6 \text{ N}$

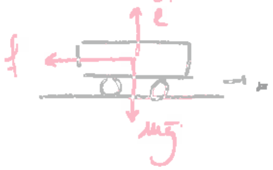
$Mg = 5{,}5 \cdot 10^5 \text{ N} < au \Rightarrow$ ok

TD1 Complément.

Ex 6 : Mvt Rectiligne sur une route horizontale.

a) 

$\vec{v_0}$ cste $\Rightarrow \sum \vec{F} = \vec{0}$

b) 

$\rightarrow$ s/Ox : $m\ddot{x} = -k\dot{x}$

$m\ddot{x} + k\dot{x} = 0$

$\ddot{x} + \frac{k}{m}\dot{x} = 0 \Rightarrow \dot{x} = v_0 e^{-\frac{kt}{m}}$

$x = x_0 - \frac{v_0 m}{k} e^{-kt/m}$

$x(t=0) = 0 \Rightarrow x_0 = \frac{v_0 m}{k}$

$x(t) = \frac{v_0 m}{k}\left(1 - e^{-kt/m}\right)$

c) {moustique + voiture} = syst. pseudo-isolé

$\frac{d\vec{p}}{dt} = \vec{0} \Rightarrow \vec{0} + m_{moustiq}\vec{v_0} = (m + m_{moustique})\vec{v_1}$

$\vec{v_1} = \frac{m_{moust}}{m + m_{moust}}\vec{v_0}$

Ex 2. Chute libre.

a) #1 : $m\ddot{z} = -mg$

$z = -\frac{1}{2}gt^2 + h_0$

#2 : $\begin{cases} m\ddot{z} = -mg \\ m\ddot{y} = 0 \end{cases}$

$\begin{cases} z = -\frac{1}{2}gt^2 + h_0 \\ y = -v_0 t \end{cases}$

b) Même hauteur à tt instant

c) Atteint le sol quand $t = \sqrt{\frac{2h_0}{g}}$ $\quad y = -v_0\sqrt{\frac{2h_0}{g}}$

Collision si la dist. $D < -v_0\sqrt{\frac{2h_0}{g}}$

d) Dirigé vers le haut

Touche le sol en 1er

~~Touche le sol en même temps.~~

$\begin{cases} \dot{z} = -gt + v_{0z} \\ \dot{y} = v_{0y} \end{cases}$

$z = -\frac{1}{2}gt^2 + h_0 + v_{0z}t$

$y = v_{0y}t$

TD 2 - Energie

Ex 9 : Quelle hauteur pour ce jet d'eau ?

Conservat° de l'énerg : $\frac{1}{2} m v^2 (t=0) = mgh$

$E_p = 0$ au niveau du lac

$v = 0$ au haut du jet

$v = \sqrt{2gh}$ ou $h = \frac{v^2}{2g}$ 200 km/h = $\frac{2.10^5}{3600}$ = 55,5 m/s

$h \approx 157 m \sim 160 m$.

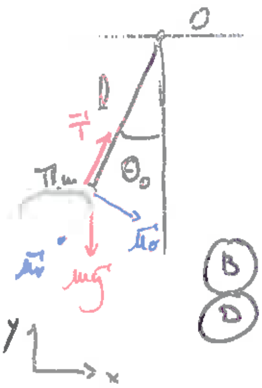
Ex 10 : Mais jusqu'où iront-ils ?

Conservat° de l'énerg $\frac{1}{2} m v^2 = mgh$

$h = \frac{v^2}{2g} = 5.1 m$.

$\sim 10 m.s^{-1}$

Ex 8 : Pendule Simple



A 1) $\vec{OM} = r\vec{u_r}$

2) $\vec{v} = l\dot{\theta}\vec{u_\theta}$

$\vec{a} = l\ddot{\theta}\vec{u_\theta} - l\dot{\theta}^2\vec{u_r}$.

$\cancel{\frac{d\vec{u_r}}{dt} = -\sin\theta\,\vec{u_x}\dot{\theta} - \cos\theta\,\vec{u_y}\dot{\theta} = \vec{u_\theta}\,\dot{\theta}}$

$\cancel{\frac{d\vec{u_\theta}}{dt} = -\cos\theta\,\vec{u_x}\dot{\theta} - \sin\theta\,\vec{u_y}\dot{\theta} = -\vec{u_r}\,\dot{\theta}}$

$\vec{u_r} = \sin\theta\,\vec{u_x} - \cos\theta\,\vec{u_y}$

$\vec{u_\theta} = \cos\theta\,\vec{u_x} + \sin\theta\,\vec{u_y}$

$\frac{d\vec{u_r}}{dt} = \dot{\theta}(+\cos\theta\,\vec{u_x} + \sin\theta\,\vec{u_y}) = \dot{\theta}\vec{u_\theta}$

$\frac{d\vec{u_\theta}}{dt} = -\dot{\theta}\vec{u_r}$

B D $m l\ddot{\theta}\vec{u_\theta} - m l\dot{\theta}^2\vec{u_r} = -T\vec{u_r} + mg\cos\theta\,\vec{u_r} - mg\sin\theta\,\vec{u_\theta}$

$l\ddot{\theta} = -g\sin\theta \xrightarrow{\text{pts angles}} l\ddot{\theta} + g\theta = 0$

$\ddot{\theta} + \frac{g}{l}\theta = 0$

$\theta = \theta_0\left[\cos\left(\sqrt{\frac{g}{l}}\,t\right) + \sin\left(\sqrt{\frac{g}{l}}\,t\right)\right]$

$\theta(t=0) = \theta_0$.

$\theta = \theta_0 \cos\sqrt{\frac{g}{l}}\,t$

C $E_m = \frac{1}{2} m l^2\dot{\theta}^2 - mgl\cos\theta$ (ref E_p en O).

$\frac{dE_m}{dt} = 0 = \cancel{m} l^{\cancel{2}}\cancel{\dot{\theta}}\ddot{\theta} + g\cancel{l}\sin\theta\,\cancel{m\dot{\theta}} \Rightarrow \ddot{\theta} + \frac{g}{l}\sin\theta = 0$