TD3 - Forces Centrales.

Ex 1.

1) $E_p = -\frac{k m_1 m_2}{r} + cste.$

2) Non.

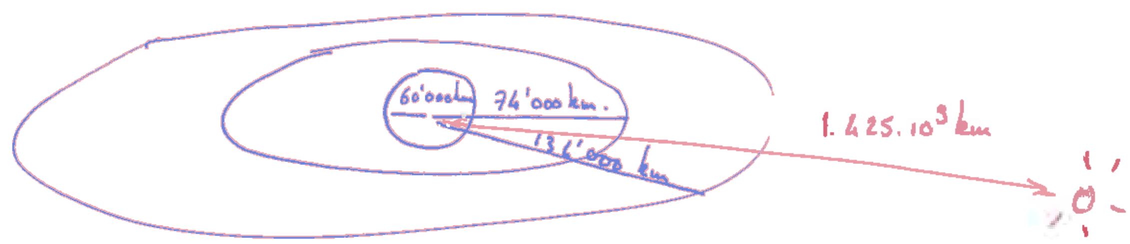
3) = 1)

4) $\vec F = q v B \vec{u_r}$ $\longrightarrow$ ~~Ep = ...~~ Non.

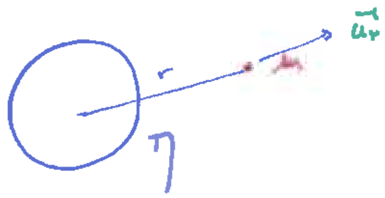
5) Non.

6) $\vec F = q r B \vec{u_r}$ $\longrightarrow$ $E_p = -\frac{1}{2} q r^2 B + cste$

Ex 2



1) L'interaction de gravitat°



$$\vec F = -\frac{m M G}{r^2}\vec{u_r}.$$

2) Satellite gravitant s/ une orbite circul°.

(a) Réf. saturnocentriq. : centre = centre de Saturne.
axes = vers étoiles fixes.

(b) $\vec a = -\frac{MG}{r^2}\vec{u_r} = -\frac{v^2}{r}\vec{u_r}$ $\Big\}\longrightarrow$ $\|v\| = \sqrt{\frac{MG}{r}} = cste.$

(c) Temps pr effectuer une orbite. $v = \frac{2\pi r}{T}$ => $T = 2\pi r\sqrt{\frac{r}{MG}}$

↳ $T^2 = \frac{4\pi^2 r^3}{MG}$ 3e loi de Kepler.
cste de la loi Kepler

$$M = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2 \times (159.10^6)^3}{6,67.10^{-11}\times(17\times3600+58\times60)^2} = 5.7.10^{26}\ kg.$$

3) Dispositif d'une série d'objets ponctuels s/ une même orbite.

Ils ont tous la même vitesse => se déplacent collectivement.

4) Dispositif de 2 objets s/ 2 orbites ≠.

$T = 2\pi r\sqrt{\frac{r}{\eta G}}$ => $r \nearrow$ $T \nearrow$ => A en retard s/B.

config. 2.

5) Anneaux de Saturne.

$\vec{\eta}\ \|\vec{r}\|$ => $\vec{m}\ \|\vec{v}\|$ si un anneau.

Anneaux ont vit. ≠.

=> vit. ≠ => f. de cisaillement => auraient dû exploser.

$\|\vec{v}\| = \sqrt{\frac{\eta G}{r}}$ ↗ qd r ↘

6) La sphère de Roche.

• $F_{AB} = \frac{m^2 G}{4\rho^2}$

$F_{S/A} = \frac{m\eta G}{(r-\rho)^2}$

$F_{S/B} = \frac{m\eta G}{(r+\rho)^2}$

$$F_{S/A} - F_{S/B} = m\eta G\left[\frac{1}{(r-\rho)^2} - \frac{1}{(r+\rho)^2}\right].$$

ordre 2

car $\rho \ll r$

$$\frac{4r\rho}{(r-\rho)^2(r+\rho)^2} \sim \frac{4r\rho}{r^4}$$

$$F_{S/A} - F_{S/B} = 4m\eta G\frac{\rho}{r^3}$$

marées.

• R_0 / $\frac{m^2 G}{4\rho^2} \sim 4 m\eta G\frac{\rho}{R_0^3}$ => $R_0 = \sqrt[3]{\frac{16\eta}{m}}\,\rho$

$r > R_0$ F_{AB} gagne => gros objet

$r < R_0$ F_{AB} perd => [illegible]

Satellites → au-delà de R_0.

Exercice 3 - Orbite de la Lune & Terre.

1) $\vec{OM} = r\,\vec{u_r}$ $\quad \vec{L} = m\,\vec{r} \wedge r\dot{\theta}\,\vec{u_\theta} = mr^2\dot{\theta}\,\vec{u_z}$.

$\vec{v} = R\dot{\theta}\,\vec{u_\theta}$

$\vec{a} = -R\dot{\theta}^2\,\vec{u_r}$.

2) $E_c = \frac{1}{2} mR^2\dot{\theta}^2$

$E_p = -\frac{M_T m G}{R}$

$\Bigg\}\ E_m = \frac{1}{2} mR^2\dot{\theta}^2 - \frac{M_T m G}{R}$

3) $E_m = \frac{L^2}{2mR^2} - \frac{M_T m G}{R} = \frac{G M_T m^{\cancel{2}} \cancel{R}}{2\cancel{m} R^{\cancel{2}}} - \frac{M_T m G}{R}$

$E_m = -\frac{m M_T G}{2R}$

4) $\mathcal{A} = \frac{L_0}{2m} T = \pi R^2$ $\quad T = \frac{2\pi R^2}{\,} \sqrt{\frac{1}{G M_T \cancel{m} R}} = \sqrt{\frac{4\pi^2 R^3}{G M_T}} = T$

5) $\frac{T^2}{R^3} = \frac{4\pi^2}{G M_T}$ indépendant de m.

$E_m = \frac{1}{2} mR^2\dot{\theta}^2 - \frac{m M_T G}{R}$

$L = mR^2\dot{\theta}$

$L^2 = m^2R^4\dot{\theta}^2$

$\frac{L^2}{R} = m^2R^3\dot{\theta}^2$

$L^2 = (mr^2\dot{\theta})^2 = m^2R^4\dot{\theta}^2$

$E_c = \frac{1}{2} mR^2\dot{\theta}^2$

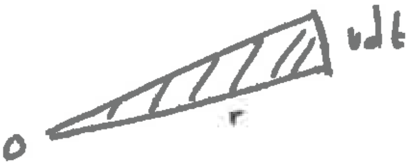
$E_p = -\frac{M_T m G}{R}$

$\Big)\ E_m = \frac{1}{2} mR^2\dot{\theta}^2 - \frac{m M_T G}{R}$

$\Delta E_m = 0 \Rightarrow E_m = \text{cte}$

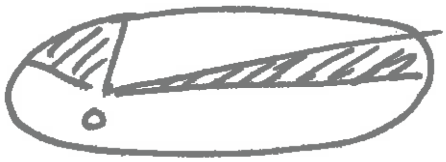
$E_m = -\frac{1}{2}\frac{G M_T M_L}{R} = -\frac{G M_T M_L}{R} + E_c$. $\quad E_c = \frac{1}{2}\frac{G M_T M_L}{R} = -\frac{1}{2} E_p$

3. Ex d'applications.



- Aire balayée pendant dt.

$$d\mathcal{A} = \frac{1}{2} r v dt = \frac{1}{2} r^2 \dot{\theta} dt = \frac{L_0}{2m} = cste$$



L'aire balayée pdt dt est tjs la même ss intervalle $\neq$ grde.

Ppté 4 : Conservat° de l'Em.

- Thm l'Em : $\Delta E_m = W_{f.non\ cons.} = 0$ en l'abs. d'autres forces.

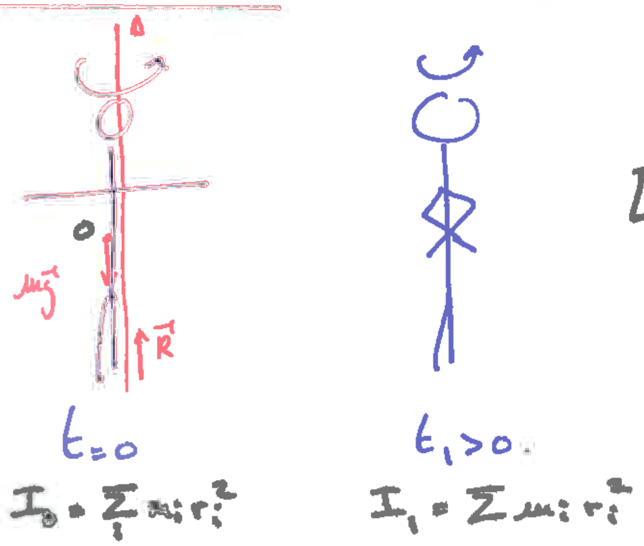
- $E_m = \frac{1}{2} m v^2 + E_p = cste.$

$$= \frac{1}{2} m (r\dot{\theta} \hat{u_\theta} + \dot{r} \hat{u_r})^2 - \frac{A}{r}$$

$$E_m = \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{1}{2} m r^2 \dot{\theta}^2 - \frac{A}{r}}_{E_{p\,eff}}.$$

pour étudier traj. possibles :
- paraboles
- hyperboles.
- cercles
- ellipses.

III Patineur.



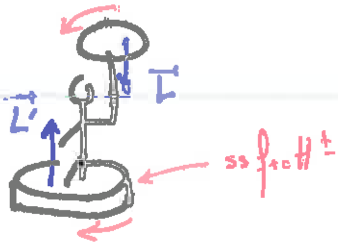
$$\frac{d\vec{L_0}}{dt} = \vec{\mathcal{M}}_{\vec{g},O} + \vec{\mathcal{M}}_{\vec{R},O} = \vec{0}.$$

$$\vec{L_0} = I \omega \vec{u_z} = cste$$

$$I_0 \longrightarrow I_1 < I_0$$
$$\omega_0 \longrightarrow \omega_1 > \omega_0$$

La vitesse de rotat° ↗

IV Gyroscope (1)



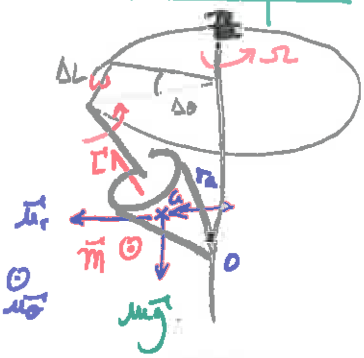
$t = 0$

$L_z = 0$

t_1

$\frac{d\vec L}{dt} = \vec 0$ => la chaise tourne dans la direct° opposée pour compenser

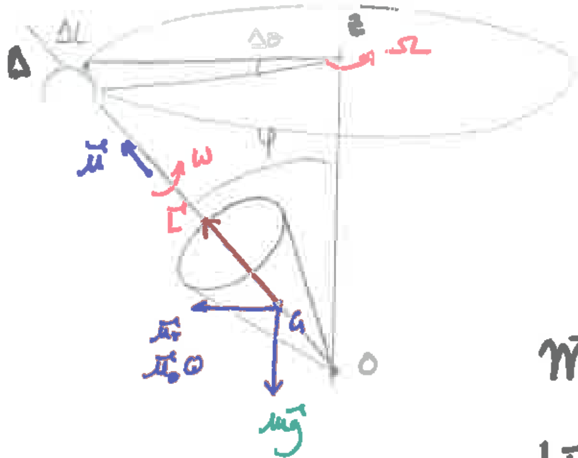
V Toupie.



~~Le poids exerce un mt :~~

$\cancel{\vec{\mathcal{M}}_{\vec P, z} = r_G \vec u_r \wedge m\vec g = m r_G g \vec u_r (\vec u_z)}$

$\cancel{= m r_G g \vec u_\theta = \frac{d\vec L}{dt}}$



• La toupie a un mt cinétique $\vec L = I\omega\vec u$ // Δ.
(solide en rotat°)

• Le poids exerce un mt / O

$\vec{\mathcal{M}}_{\vec P, O} = r_G \vec u \wedge m\vec g = m g r_G \sin\varphi \, \vec u_\theta$

$\frac{d\vec L}{dt} = m g r_G \sin\varphi \, \vec u_\theta$

Or $\Delta L = \Delta\theta \, L \sin\varphi$ => $\Delta\theta = \frac{\Delta L}{L\sin\varphi}$ => $\Omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta L}{\Delta t}\frac{1}{L\sin\varphi}$

$\Omega = \frac{m g r_G}{I\omega}$. qd r_G ↗ (toupie tombe) => Ω ↗.