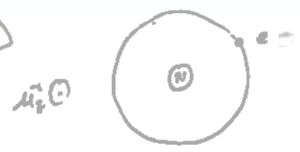
TD n°2

Exercice 3 - Moment cinétique :

1)

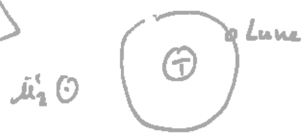


$\vec{L_o} = m\vec{r}\wedge\vec{v} = m r^2\dot{\theta}\,\vec{u_z} = \underline{1.06.10^{-34}\,\vec{u_z}}$ [$kg\,m^2\,s^{-1}$].

$f = 6.6.10^{15}$ Hz $\Rightarrow$ Parcourt 2π en $1.51.10^{-16}$ s.

$\Rightarrow \omega = 4.14.10^{16}$ rad.s^{-1} $= \dot{\theta}$

2)



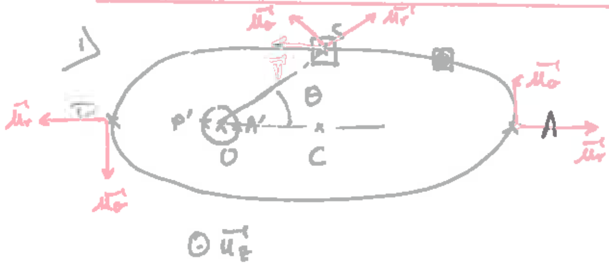
$\vec{L_o} = m r^2\omega\,\vec{u_z} = m r^2\frac{2\pi}{T}\,\vec{u_z} = \underline{2.81.10^{34}\,\vec{u_z}}$ [$kg\,m^2\,s^{-1}$]

28/34

⚠ r en m.

Exercice 4 - Vitesse d'un satellite sur son orbite elliptique :

1)



$48'150/2 = 24'075$ km.

2) $\vec{L_o} = m\vec{r}\wedge\vec{v} = m r\,\vec{u_r}\wedge(-v_s\cos\theta\,\vec{u_r} + v\sin\theta\,\vec{u_\theta})$.

$= m r\,v_s\sin\theta\,\vec{u_z}$.

$= m \cancel{r}\,v_s\,\frac{SC}{\sqrt{OC^2+SC^2}} = \underline{6.08.10^{13}\,\vec{u_z}}$ [$kg\,m^2\,s^{-1}$].

Or $OC = OA' + A'C = OA' + A'A - AC = 17'325$ km.

$\Rightarrow OS = 24'074$ km.

et $v_s = 14650$ km/h $= 4069$ m/s.

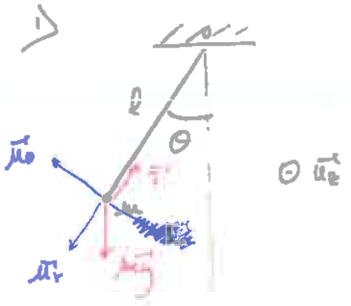
3) Conservat° du mt cinétiq :

<u>En P</u> $\vec{L_o} = m\,\underset{\downarrow\ 6750\,km}{OP}\,\vec{u_r}\wedge v\,\vec{u_\theta} \Rightarrow v = \frac{\|\vec{L_o}\|}{m\,OP} = 6'996$ m/s $= \underline{25'185 \text{ km/h}}$.

<u>En A</u> $v = \frac{\|\vec{L_o}\|}{m\,\underset{\downarrow\ 41'400\,km}{OA}} = 1'135$ m/s $= \underline{4'087 \text{ km/h}}$.

Exercice 4 : Pendule simple

1)



2) $\vec{OM} = r\vec{u_r}$

3) $\vec{\Gamma}_{m\vec g,O} = \vec r \wedge \vec F = r\vec{u_r} \wedge \left(mg\cos\theta\,\vec{u_r} - mg\sin\theta\,\vec{u_\theta}\right)$

$= -rmg\sin\theta\,\vec{u_z} \sim -rmg\theta\,\vec{u_z}$

4) $\vec L = m\vec r \wedge \vec v = mr^2\omega\,\vec{u_z}$

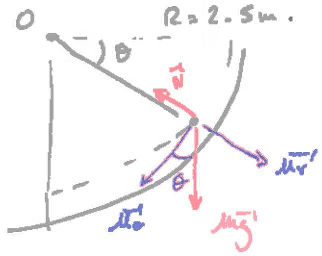
5) $\frac{d\vec L}{dt} = \vec{\Gamma}_{m\vec g,O} \Rightarrow -rmg\theta = mr^2\ddot\theta + \cancel{2mr\dot\theta}\,\dot r$, $\dot r = 0$

$\ddot\theta = -\frac{g}{l}\theta \Rightarrow \ddot\theta + \frac{g}{l}\theta = 0$

$\theta = \theta_0 \cos\sqrt{\frac{g}{l}}\,t$

Exercice 5 : Vitesse d'un enfant à la sortie d'un toboggan

1)



2) $\frac{d\vec{L_O}}{dt} = \vec{\Gamma}_{m\vec g,O}$

$\vec{L_O} = m r\vec{u_r} \wedge r\dot\theta\,\vec{u_\theta} = mr^2\dot\theta\,\vec{u_z}$

$\vec{\Gamma}_{m\vec g,O} = \vec r \wedge \vec F = r\vec{u_r} \wedge mg\cos\theta\,\vec{u_\theta} = mgr\cos\theta\,\vec{u_z}$.

$\cancel{m}r^{\cancel{2}}\ddot\theta = \cancel{m}g\cancel{r}\cos\theta$

$\ddot\theta = \frac{g}{OG}\cos\theta$

3) $\dot\theta\ddot\theta = \frac{g}{OG}\dot\theta\cos\theta$

$\frac{d\dot\theta^2}{dt} \times \frac{1}{2} = \frac{g}{OG}\frac{d}{dt}(\sin\theta) \Rightarrow \dot\theta^2 = \frac{2g}{OG}\sin\theta + \text{cte}$

à $t=0$ $\quad 0 = \frac{2g}{OG}\sin\theta_0 + \text{cte}$

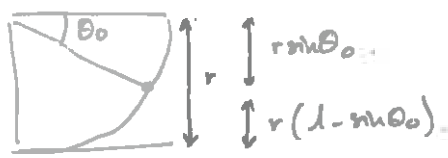
$\dot\theta^2 = \frac{2g}{OG}\left(\sin\theta - \sin\theta_0\right)$

$v = r\dot\theta = \sqrt{2gr\left(\sin\theta - \sin\theta_0\right)}$

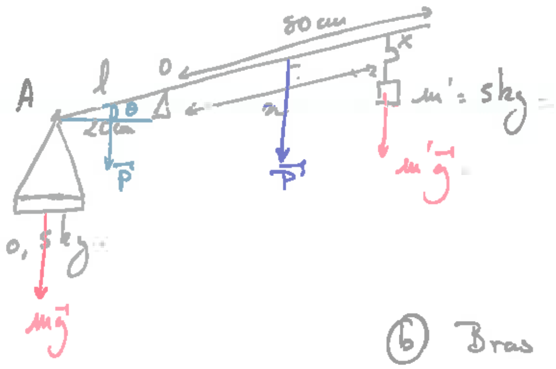
4) Vitesse max en $\theta = \pi/2$

$v = \sqrt{2gr\left(1 - \sin\theta_0\right)} = 6$ m/s

Ce que l'on retrouve via conservat° de l'éng.



Exercice 6 : Balance.



1) a) $mgl = m'gx_0 \Rightarrow x_0 = \frac{m}{m'} l = 2$ cm

b) $(m+\Pi)gl = m'gx \Rightarrow x = \frac{m+\Pi}{m'} l = 42$ cm.

2) a) $\mu = 5 \text{ kg.m}^{-1} = 0,05 \text{ kg.cm}^{-1} = \rho \times S$

$\Rightarrow S = 6,35 \text{ cm}^2$

b) Bras droit : 1kg.
Bras gauche : 4kg.

$$mgl + g\frac{l}{2} = 4g\times 0,4 + m'gx_0$$

$$x_0 = \frac{(m+1/2)l - 4\times 0,4}{m'} = -28 \text{ cm vers } A.$$

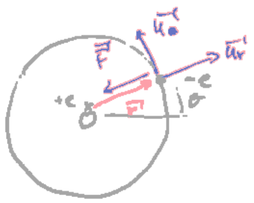
c) $(m+\Pi)gl + g\frac{l}{2} = 4g\times 0,4 + m'gx$

$$x = \frac{(m+\Pi+1/2)l - 1.6}{m'} = 12 \text{ cm} = x$$

d) $\Pi = \frac{4\times 0.4 + m'x - l/2}{l} - m = 17.5 \text{ kg} = \Pi$

$\Rightarrow$ c'est dommageable.

Exercice 8 : Atome de Bohr.



1) Classiq : masse ponctuelle.

2) $\vec F = \frac{-e^2}{4\pi\varepsilon_0}\frac{\vec r}{r^3}$

3) $\vec\Gamma_{F,O} = \vec F \wedge \vec F = \vec 0 \Rightarrow$ rien ne fait tourner l'e⁻ de l'autre plan.

4) $E_p = -\frac{k}{r}$

5) $\vec L = m r \vec{u_r} \wedge r\omega \vec{u_\theta} = mr^2\omega \vec{k} = \vec L$

6) $\vec{OM} = r\vec{u_r}$.

$\vec v = r\dot\theta \vec{u_\theta}$

$\vec a = r\ddot\theta \vec{u_\theta} + r\dot\theta \frac{d\vec{u_\theta}}{dt} = r\ddot\theta \vec{u_\theta} - r\dot\theta^2 \vec{u_r}$

$\vec{u_\theta} = -\sin\theta\, \vec{u_x} + \cos\theta\, \vec{u_y}$

$\frac{d\vec{u_\theta}}{dt} = \dot\theta(-\vec{u_r})$

a) $m(r\ddot\theta \vec{u_\theta} - r\dot\theta^2 \vec{u_r}) = -\frac{k}{r^2}\vec{u_r} \Rightarrow \ddot\theta = 0 \Rightarrow \dot\theta = \text{cste} \Rightarrow$ unif.

b) $\|\vec v\| = \sqrt{\frac{k}{mr}}$

c) $E_m = \frac{1}{2} m\frac{k}{mr} - \frac{k}{r} = -\frac{k}{2r} = E_p$

7) $\|\vec{L}\| = m r^2 \omega$ et $v = r\dot{\theta} = \sqrt{\frac{k}{mr}} \Rightarrow \dot{\theta} = \sqrt{\frac{k}{mr^3}}$

$$\|\vec{L}\| = \sqrt{\frac{m^2 r^4 k}{m r^3}} = \sqrt{mrk} = \|\vec{L}\|$$

8) $\sqrt{mrk} = \hbar n \rightarrow R = \frac{\hbar^2 n^2}{mk}$; $R_B = \frac{\hbar^2}{mk} = 0{,}052 \text{ nm}$.

$$E = -\frac{k}{2r} = \frac{-k^2}{2\hbar^2 n^2} m \quad ; \quad E_B = \frac{k^2 m}{2\hbar^2} = 13{,}8 \text{ eV}.$$

Exercice 8 – Spectro de masse.

1) (a) $m\vec{a} = q\vec{E}$.

(b) $m\dot{x} = qEt$

$$\begin{cases} x(t) = \frac{1}{2}\frac{qE}{m}t^2 \\ y(t) = 0. \end{cases}$$

2) (a) $m\vec{a} = q\vec{v}\wedge\vec{B} = q\dot{x}\,\vec{u_x}\wedge B\vec{u_z} + q\dot{y}\,\vec{u_y}\wedge B\vec{u_z}$.

$= -q\dot{x}B\vec{u_y} + q\dot{y}B\vec{u_x}$

$$\begin{cases} \ddot{x} = \frac{qB}{m}\dot{y} \\ \ddot{y} = -\frac{qB}{m}\dot{x} \end{cases} \qquad \begin{cases} \dot{V_x} = \frac{qB}{m}V_y = \omega V_y \\ \dot{V_y} = -\frac{qB}{m}V_x = -\omega V_x \end{cases} \ (*)$$

(b) (c) $\begin{cases} \ddot{V_x} = -\omega^2 V_x \longrightarrow V_x(t) = A\cos(\omega t) + B\sin(\omega t) = V_0\cos(\omega t). \\ \ddot{V_y} = -\omega^2 V_y \longrightarrow V_y(t) = B\sin(\omega t) = V_0\sin(\omega t) \text{ via } (*) . \end{cases}$

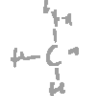
(d) $\begin{cases} x(t) = \frac{V_0}{\omega}\sin(\omega t) \\ y(t) = -\frac{V_0}{\omega}\cos(\omega t). \end{cases}$

(e) $x^2 + y^2 = \frac{V_0^2}{\omega^2} \Rightarrow$ cercle de centre O et $R = \sqrt{\frac{V_0^2}{\omega^2}} = \frac{V_0}{\omega}$ [m]

3) (a) Appliquer $\vec{E}$.

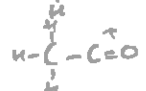
(b) Rayon dépend de $R = \frac{V_0 m}{qB} \longrightarrow$ + légers, + lourds out.

(c) $\frac{m}{z} = 15 \Rightarrow CH_3^+$

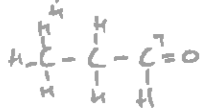


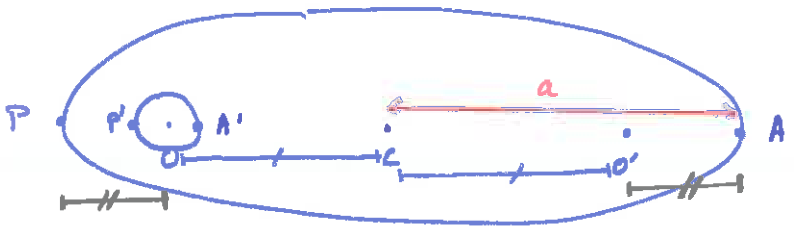
4) ?

$\frac{m}{z} = 43 \Rightarrow C_2H_3O^+$



$\frac{m}{z} = 57 \Rightarrow C_3H_5O^+$





Les foyers O et O' d'une ellipse sont tels que tout point M appartenant à l'ellipse vérifie :

$$OM + O'M = 2a \quad \text{où } a = \tfrac{1}{2} \text{ grand axe}$$

On applique cette propriété aux points P et A, d'où :

$$OP + O'P = OA + O'A = 2a.$$

Ainsi $OP = 2a - O'P = 2a - (AP - O'A) = 2a - 2a + O'A$

D'où $\underbrace{\boxed{OP = O'A}}_{(1)}$ et donc $\underbrace{\boxed{OC = O'C}}_{(2)}$.

Alors $OC \underset{(2)}{=} \frac{1}{2}(OC + CO') = \frac{1}{2}(2a - O'A - OP) = \frac{1}{2}(PA - O'A - OP)$

$$= \frac{1}{2}(PO' - OP) \underset{\substack{(1)\text{ et }(2)\\ \text{donnent } O'P = OA}}{=} \frac{1}{2}(OA - OP)$$

$$= \frac{1}{2}(OA' + A'A - OP' - P'P) \underset{OP' = OA' = R_T}{=} \frac{1}{2}(A'A - P'P).$$