PARTICLE ACCELERATORS : CONTROL & OPTIMIZATION PROBLEM FOR POWER SOURCES

JEAN-LUC BABIGEON[1]

[1]**Laboratoire de l'Accélérateur linéaire** (CNRS – LAL/DACC), Centre scientifique d'Orsay - 91898 Orsay - France

Summary

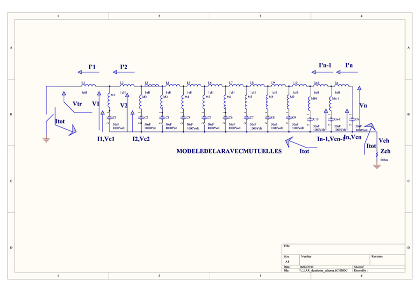
Particle physics in Large Hadron Collider LHC at CERN needs low emittance beams for reaching a minimum of energy/position dispersion of colliding bunches. For that, the High pulsed power sources, like High Voltage modulators must deliver an electrical, ideally "flat top" (inside that plateau, the energy of emitted particles is constant). To optimize that flat top, we need to tune some passive Pulse Forming Network PFN. The problem of optimal tuning lies in "control and optimization" class. In our PFN which is 12-strongly coupled harmonic oscillator, it depends on 12 continuous variables, who are the values of tunable power inductances. The optimal tuning state is not described by a trivial algorithm. What we need is to find a control and optimization method, either by analytical -if possible- approach, either by analysis of measurement during tuning. As classically, the time or frequency waveforms do not help us to improve the tuning, essentially because in time measurements, 2 distant cells of PFN may interfere with unknown phases. We then performed an analysis by complex wavelet.
In alternative approach, we proposed to solve the correspondent evolution equation, which belongs to Hill family, because it is linked with a pseudo transmission line with inhomogeneous characteristic impedance. We have extrapolated the Flocquet method by infinite determinants, but in synthesis-like algorithm.
Inside the same problem, we raise another different question on the topic of mixed/intricate time frequency analysis : is there a class of transform which would use a "mixed variable" which we call for example w. For sake of simplicity, w is complex and we have w= g(f,t). All our measurement V(t) or V(f) should be then transformed/mapped to V(w) where V is the High voltage image of the power source.
To make our question or formalism useful, we have to restrict the features of g. That discussion and these ideas could help us in many other topics of the physic, like Electromagnetic field measurements (photo emission assisted by Schottky effect) for femtosecond electrons bunches. That issue could also be beneficial in other scientific researches.

I Flat top waveform for RF acceleration

1. Description of experiment

In electrons Linear Accelerators for particle physics, e- bunches are today accelerated by Radio Frequency (GHz) modulated electric fiels ; these fields are generated by power RF klystrons, typically pulses of 200kV/200A peak/5μs at PHIL Accelerateur - Orsay -France. Example of a power waveform hereafter. In fact, the idealistic flat top reveals irregularities, and the principal cause comes from the tuning of power source, particularly the Delay Line or Pulse Forming Network ([1]). The PFN, is displayed below. It is made of a passive system of 12 LC cells, like a pseudo transmission line ; the capacitors C, of same value 50nF, are charged to a High Voltage, and a power switch generates a voltage waveform front in that line.

Voltage pulsed measurement, example



PFN Schematic

You can see a photo of installation inside the power modulator of the PHIL Accelerator, The inductances L are continuously tunable, in the $[0, 5\mu H - 1\mu H]$ range ; hence there is a correspondance between the 12-set of values and the behaviour of waveform. For a mean value of inductance, there exists a tuning which optimize the flat top.

Photo of installation

here starts the control problem, ie **"to determine the optimal tuning (a set of values L1, ..L12), with the sole observation of the ripples in the voltage and/or the current waveform(s)"**.

2. Firsts treatments by Frequency analysis

* The PFN constitutes a 12 bodies strongly coupled. Considered as a filter, the PFN frequency transfer functions are polynomials of degree 12 for the complex frequency variable. Its Cauer decomposition :

$$Z(p) = L_1 p + \cfrac{1}{C_1 p + \cfrac{1}{L_2 p + \cfrac{1}{C_2 p + \cfrac{1}{L_3 p + \cfrac{1}{C_3 p \ldots + \cfrac{1}{C_{n-1} p + \cfrac{1}{L_n p + \frac{1}{C_n p}}}}}}}} \qquad (1)$$

is not tractable to obtain even an analytical form of an eigen frequency, set by the parameters L1,12. or to transform it into Foster form with poles and zeros ([4]).

* The Kirchhoff direct treatment of that network brings us -in frequency domain- to an expression of impedance matrix ([1]) The solution of that system needs the determination of the currents I_j, hence their inversion to the time space, this implies the analysis of the matrix K. Different methods were tried, unsuccesfully, to solve it : Jordan decomposition, time Form of K, expliciting the frequency operators as time one, analysis of K in distincts frequency ranges.

Explanation of failure : we have to solve a determinant of degree 12.

II Hill equation for transmission lines

Each couple L_iC is viewed as an infinitesimal element of transmission line of characteristic impedance $\sqrt(\frac{L_i}{C})$. Problem : mastering of the 12 continuous varying L_i to reach that optimal tuning. Hill equation for inhomogeneous transmission line [2], [1] :

$$\ddot{\psi(x)} + M(x)\psi(x) = 0, \text{ with } \psi(x) = \begin{pmatrix} u(x) \\ i(x) \end{pmatrix} \text{ and } M(x) = \begin{pmatrix} M_z(x) & 0 \\ 0 & M_y(x) \end{pmatrix} \qquad (2)$$

with
$u(x) = \sqrt(Z(x))V(x)$, $i(x) = \sqrt(Y(x))I(x)$, $Y(x)Z(x) = Y(x) \times YZ_c^2 = Y^2Z_c^2 = G(x)$, $M_b = \frac{\ddot{\sqrt{b}}}{\sqrt{b}} - \frac{\dot{b}\dot{\sqrt{b}}}{b\sqrt{b}} - YZ$, **b=Z(x) or Y(x)**, Y(x), Z(x) admittance and impedance of the line, function of x abscissa, u(x) and i(x) voltage and current "scaled" by $\sqrt(Z(x))$and$\sqrt(Y(x)$ respectively ($\dot{}$ is the x-derivation. Slightly transformed, M_b, comes to $M_b = \frac{b''}{b} + YZ$: double Hill equation on the ψ vector.

III Direct Résolution

1. Development of M in conjugate variable

Periodization of M(x) and to decomposition in Fourier serie.

$$J_b(x) = \sum_{m=-\infty}^{+\infty} \hat{J}_{b,m} e^{jmkx} \text{ with } \hat{J}_{b,m} = \frac{1}{T}\int^{+T} J_b(x) e^{jmkx} dx \qquad (3)$$

k : x conjugate variable, $\hat{J}_{b,m}$: m-Fourier coefficient of J_b, periodization of M. Solutions like :

$$\psi(x) = D_1 e^{\mu_1 x}\psi_1(x) + D_2 e^{\mu_2 x}\psi_2(x)$$

μ_i : Flocquet exponents for $i = 1, 2$ and $\psi_i(x)$ are periodic (state) vector of periode π et D_i are 2 constants.
i(x) linked to I(x) by $i(x) = \sqrt(Y(x))I(x)$and$I(x) = \frac{1}{Z(x)} \times -\frac{dV(x)}{dx}$, The number of exponents is related to the number of degrees of freedom. Here the independant 1-dimensional state variables are 2, i and v, so n=2.
If these solutions are decomposed in Fourier series, We have to the (double) infinite system of equations,

$$(\mu + jnk))^2\psi_n + \sum_{p=-\infty}^{+\infty} \hat{J}_{b,n-p} \times \psi_p = 0 \qquad (4)$$

so, dividing by the square quantity $(\mu + jnk))^2$, we have $D(\mu) \times \psi = 0$, with :

$$D_\mu = \begin{pmatrix} \hat{J}_{z,n-p} & 0 \\ 0 & \hat{J}_{y,n-p} \end{pmatrix} \qquad (5)$$

Practically, that infinite déterminant is troncated to an adequate order, dependant to the desired precision .
Our -inverse-problem : knowing behavior of solutions by measurements, to determine the function $J(x)$, and so $Z(x)$,$Y(x)$. Last, $Z(x)$ is the image of the actual tuning set responsible of the form of the waveforms. We have to invert the relation $D(\mu) \times x = 0$, to find $D(\mu)$.

2. Resolution of inverse problem

For example, truncation to second order gives for the first block dedicated to the u-component of ψ :

$$\begin{pmatrix} d_{\mu,-2} & \hat{J}_{z,-1} & \hat{J}_{z,-2} & \hat{J}_{z,-3} & \hat{J}_{z,-4} \\ \hat{J}_{z,1} & d_{\mu,-1} & \hat{J}_{z,-1} & \hat{J}_{z,-2} & \hat{J}_{z,-3} \\ \hat{J}_{z,2} & \hat{J}_{z,1} & d_{\mu,0} & \hat{J}_{z,-1} & \hat{J}_{z,-2} \\ \hat{J}_{z,3} & \hat{J}_{z,2} & \hat{J}_{z,1} & d_{\mu,1} & \hat{J}_{z-1} \\ \hat{J}_{z,4} & \hat{J}_{z,3} & \hat{J}_{z,2} & \hat{J}_{z,1} & d_{\mu,2} \end{pmatrix} \qquad (6)$$

where $d_{\mu,n} = (\mu + jnk)^2 + \hat{J}_0$ and $\mu_{0i} = j\sqrt(\hat{J}_0)$. When we scale by $d_{\mu,n}$, droping z index we obtain for one solution,

$$\begin{pmatrix} 1 & \frac{\hat{J}_{-1}}{d_{-2}} & \frac{\hat{J}_{-2}}{d_{-2}} & \frac{\hat{J}_{-3}}{d_{-2}} & \frac{\hat{J}_{-4}}{d_{-2}} \\ \frac{\hat{J}_1}{d_{-1}} & 1 & \frac{\hat{J}_{-1}}{d_{-1}} & \frac{\hat{J}_{-2}}{d_{-1}} & \frac{\hat{J}_{-3}}{d_{-1}} \\ \frac{\hat{J}_2}{d_0} & \frac{\hat{J}_1}{d_0} & 1 & \frac{\hat{J}_{-1}}{d_0} & \frac{\hat{J}_{-2}}{d_0} \\ \frac{\hat{J}_3}{d_1} & \frac{\hat{J}_2}{d_1} & \frac{\hat{J}_1}{d_1} & 1 & \frac{\hat{J}_{-1}}{d_1} \\ \frac{\hat{J}_4}{d_2} & \frac{\hat{J}_3}{d_2} & \frac{\hat{J}_2}{d_2} & \frac{\hat{J}_1}{d_2} & 1 \end{pmatrix} \times \begin{pmatrix} u_2^1 \\ u_1^1 \\ u_0^1 \\ u_1^1 \\ u_2^1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (7)$$

The unknowns are the $\hat{J}_b(x)$ coefficients. Evaluation gives for that truncation, 9x2+2=18 unknowns for $\hat{J}_b$ ie 20 in total. The inputs are 10, from ψ. The reolution in general case needs to link the $\hat{J}_z$ with $\hat{J}_y$. then the characteristic impedance and the propagation exponent must be known. Nevertheless in our spêcific case, the admittance is constant, so all Fourier coefficients of $\hat{J}_y$ vanish except $\hat{J}_{y0}$, and the number of unknowns lowers to 10 in the above truncation, which is equal to the number of inputs. It is then possible to solve the linear system, of course in any higher order. A preliminar step is to determine the 2 Flocquet exponents. It is done in an analog method to [2]. Resolution of the system gives us the behaviour of $Z(x)$, them the state of tuning of the 12 inductances.
Our measurements lie in time domain, so with a proper Fourier transformation, we can extract complex coefficients (module and phase). It is important to get the complex information for the resolution of the system.

Références

[1] JL Babigeon. Remarques sur le fonctionnement et le réglage d'une ligne à retard. Technical report, CNRS/Lal/Dacc, september 2011.

[2] Mohamed Boussalem. *Etude et modelisation de structures de transmission non uniformes, application à l'adaptation d'impédance et au filtrage*. PhD thesis, july 2007.

[3] Liu. Scilab wavelet toolbox.

[4] RM Roark. Synthesis of pulse forming network for general resistive loads. Master's thesis, Texas Tech University, may 1979.

[5] Bruno Torrésani. *Analyse continue par ondelettes*. CNRS editions, 1995.

IV WAVELET DECOMPOSITION

1. Motivation & tools

1. To allow an understanding of the phenomena,
2. to define an operational tuning approach, leading to automation of the tuning

1.1 Basis of the work

We develop f on an infinite wavelet basis ψ. $T_f(b, a)$ is the wavelet coefficient. Our signals f are voltage and current of the pulsed power source, they are compact support time functions. We work on time-scale diagrams, firstly with the toolbox of Scilab. Test of the frequency extraction by 2 criteria [5],

$$\frac{\partial\Psi(b,a)}{\partial a} = 0 \text{ and, or } \frac{\partial\Psi(b,a)}{\partial b}_{x_s(b,a)=b_0} = \frac{\phi'_\psi(0)}{a}$$

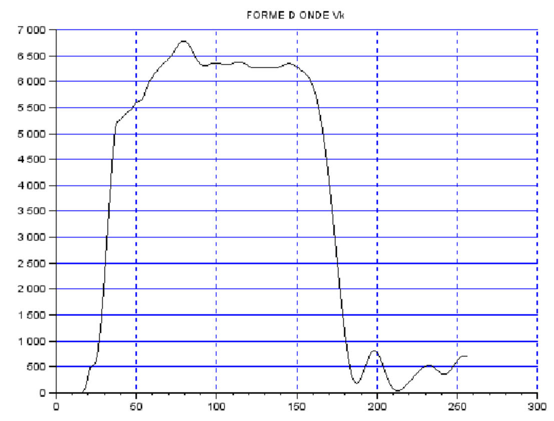
where $\Psi(b, a) = Arg(T_f(b, a))$ is the phase of wavelet coefficient. These conditions are evaluated on the edge. The choice of mother ψ functions escapes to standard "automatic " methods. Our signals are an approximations of time and band limited functions, and the noise level is not an issue.

1.2 wavelet toolbox for Scilab and curves from @Cadence/Spice

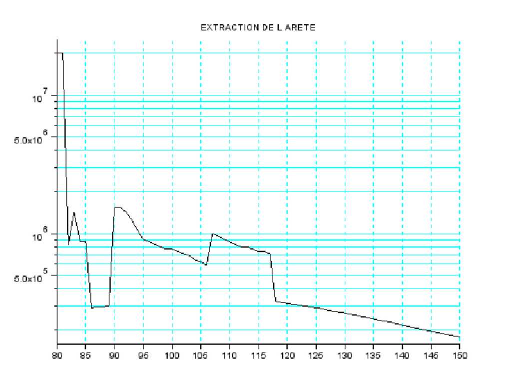
That C++ toolbox, [3], is quasi identical to @Matlab one. basic Scilab function : cwt, with syntax coef=cwt(x,scales,wname) with the standard cwplot of Scilab, and vname a list of mother wavelets. @Spice simulation represent quite correctly the phenomena. The idea is to detune artificially the PFN in @Spice, and to observe the variations of time-scale wavelets of Voltage waveform. One must define a reference tuning, for instance when the inductances has all the same value. Range of tuning is $[0.5\mu H - 1.5\mu H]$. The detuning was made on one element, then on 2 and also 3, taken in all the configurations of the 12 elements.

2. Result for calibrated detunings

Among 20 distincts simulations [1], we describe here the plots of one detuned inductance.



cellule 40 extraction de l'arête



cellule 40 extraction de l'arête

Notes :

1. lack of Scilab toolbox to parameterize correctly the range of the a scale,
2. the situation remains intricate when several inductances are detuned, particularly in opposite ranges,
3. The complex Morlet wavelet solves phase interferences for the detuned inductances,
4. The scale (frequency) parameter resolution is proportional to time resolution inside @Spice, Shannon criteria must be largely overcome (x50),
5. The criterium of phase derivative allowed again to discriminate between under and over tuning,
6. When the time resolution increases, the frequency selectivity raises also, then the scale a must be raised so that the ratio $\chi = \frac{nb_{resolutions}}{nb_{pas}}$ remain constant.

Considering the difficulty to properly scale the a parameter, I coded and praticed a complex wavelet transform. The result show the ambiguity to define the frequency of a test chirp signal ; it is central to our problem beceause of the nature of our waveforms.

V MAPPING OF TIME-FREQUENCY

1. Position of the problem

Let's a signal V(t), with such a question of optimization and control. We know that informations on time and frequency restitute, but only partially, its intricate nature, particularly the parametering. We try to define a mapping of t,f to a -for instance complex- variable w, with the relation $w = g(t, f)$, where g is a linear or not, transformation. The first questions are :

1. What are the conditions, adapted to a specific problem, of a "good" mapping ?
2. What are the best and exhaustive test function to benchmark our new transform ?
3. What are the condition of inversibility of g ?

We already know the linear transforms, the bilinear, quadratic transforms, etc... We don't here require a linear transform. but probably periodicity of the phenomena. Periodicity is either explicit, or "hidden" in physical world. So I suggest to impose a periodicity criteria to g. But the period is to be defined, we can't build a system on a period constant in time. A simple example of weakly non linear swing proves the contrary. The control and optimization problem is transposable to bifurcation theory, beceause a given non trivial tuning is not biunivocal. It can lead to multiple sets of solutions. A graphical tool will help to search a geometrical correspondance to w.

VI Conclusion

1) **The Laplace formalism led to a set of non linear equations in the frequency domain. The most general case is analyticaly unsolvable (Galois-Fubini). Finally, the frequency domain failed to describe correctly the phenomena, as the phase is not clearly showed.**

2) **Hill equation has been reviewed, linked to inhomogeneous transmission lines approximation. The determination of impedance, hence the value of tuned inductances appears tractable for the case where admittance is constant, with a variation of the infinite determinants method. The general cas requires more investigations. That step should allow us to guess a setting, starting from our voltage and current measurements.**

3) **Wavelet decomposition has been used and the first results on V curves show a possibility to localize the variation in tuning, depending of the abscissa, then for a given inductance inside the line. Here also, the improvment of analysis could lead to a better control of the precise setting. Finally, a search is suggested around a mixed variable time-frequency, which optimize the quality of temporal and fequency analysis. Some criteria are to be more defined, but being free from the standard one. Particularly, we restrict our analysis to periodic transform, but the linearity, even causality, are questionable.**